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An Economic Evaluation of Substitution in Multi-period Service and Consumable Parts Supply Chains for Low Volume, High Value Components with Dissimilar Reliability

Christopher Hertzler

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AN ECONOMIC EVALUATION OF SUBSTITUTION IN MULTI-PERIOD
SERVICE AND CONSUMABLE PARTS SUPPLY CHAINS FOR
LOW VOLUME, HIGH VALUE COMPONENTS
WITH DISSIMILAR RELIABILITY

By

Christopher Hertzler

A Dissertation
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Mississippi State University
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in the Department of Industrial Engineering and Systems Engineering

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Service parts management is an integral component of customer satisfaction. The service parts supply chain has a number of unique challenges that differentiate it from retail and manufacturing supply chains. These challenges include: unpredictable and lumpy demand, limited storage capacity, high demand service rate requirements, and high risk of obsolescence.

This research focuses on the use of substitution as a policy tool to aid in service part supply chain management; particularly with respect to low inventory and high dollar value components. In one part of this dissertation, a Markov chain is used to model unidirectional substitution with dissimilar part reliability. In addition, this work investigates probabilistic substitution policies that allow substitution to be employed on a partial basis.

This research also utilizes a Poisson process to explore steady state optimization with probabilistic substitution for a model in which a non-primary part is utilized solely as a substitute for primary parts.

The models demonstrate that both substitution protocols can significantly enhance customer performance benchmarks. Unidirectional substitution policies improve fill rate and backorder levels for the machine upon which substitution is performed. The price of this improvement is the cost of additional ordering and inventory, along with decreased fill rate and backorder performance, on the machine whose parts are used for substitution.

Substitution, using a part solely carried for that purpose, increases performance levels without higher inventory levels of either primary part. However, this type of substitution requires the inclusion of an additional inventory part and the associated costs.

Keywords: Markov chain, unidirectional substitution, service parts supply chain

DEDICATION

I would like to dedicate this dissertation to my friend Vivian without whose encouragement I would have never pursued a doctoral degree.

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CHAPTER 1

INTRODUCTION

Customer service often involves a tradeoff between cost and response time. As the customer base spreads further from the primary manufacturing and distribution centers, the cost for a given level of service grows. By reducing the cost of maintaining sufficient inventory we can increase service levels at all levels of expenditure and, in effect, provide better service for no incremental expense. As in other applications, management of the service part supply chain is a major factor in managing costs; however, the service part supply chain has a number of characteristics that differentiate it from new product supply chains. Service parts supply chains differ from other supply chain applications because when inventory shortages are encountered, the customer will experience an interruption to a working system requiring the part that is already integrated into the production process, and this shortage can cascade throughout the operation. Problems encountered include lost production, or risk of serious damage to life and health if life safety or environmental control systems are impacted.

A great deal of work has been done relating to the policy for managing inventory in the warehouse on topics such as minimizing holding cost and optimizing inventory policy. Far less work has been accomplished with respect to the type of supply chain used in remote service warehouses. In addition, the

research on substitution within the service supply chain is not mature, and there is significant opportunity for growth in the literature with respect to utilization of substitution in the service parts supply chain.

In this paper we will look at a variation of the normal inventory management policy; substitution as a method of improving parts availability and lowering cost for remote parts storage warehouses – particularly with respect to high cost components that are not stocked in large quantity. In today's increasingly global economy, firms' find themselves with significant customer installations far from the manufacturing center and primary warehouse bases. For example, an American manufacturer may have its production and distribution centers in the United States, but might have an important customer located in an area far distant from the manufacturing and storage base; for example a US firm supporting a customer in Sicily. The company may have no other customers nearby, but making sure that this customer is serviced properly is an important concern because this customer is an important component of the firm's business. As a result, it is important to make sure that parts to service this customer's machinery are available locally in order to minimize downtime.

One of the primary defining requirements for service part policy is that customers are highly intolerant of unscheduled downtime. If a component failure occurs, it is critical to bring the system back on line immediately. This means that the lead time for service parts is often very short.

Another important point of differentiation for service part inventory policy vs. standard production and inventory policy is the fact that service part demand

is often unpredictable and can be very low for a given part. This characteristic makes many statistical models for ordering, producing, and stocking of components invalid. It is not always reasonable to assume we have good information or that we have well established demand arrival distributions.

A third characteristic of service part production and inventory management policy is that the demand may be deteriorating over time, or product features may change, and this can result in an extreme risk of obsolescence. As a result, it is generally desirable to limit the amount of parts in storage or the size of production runs. In addition, given the low and unpredictable demand rate, it is not always feasible to interrupt normal production in order to produce a small product run on very short demand lead time to meet customer demand for service parts. Finally, as many of these service facilities are located in countries far distant from the factory, the shipping times and customs clearance times are not insignificant and introduce a great deal of variability into supply lead times. The end result of these factors is that service part inventories are typically small and it is not safe to assume that we can rely upon the factory or suppliers to immediately produce a replacement component.

The three traits we have mentioned create a scenario where substitution of one component for another component is very attractive. The use of substitution can reduce inventory levels and lead times while increasing service fill rates. The utilization of substitution for these purposes is the focus of this research.

The information in this dissertation is presented in the following fashion. First, we review the state of the literature for the service parts supply chain. Next we examine the literature of substitution. As we do so, we emphasize the progressive contributions and areas of focus in the existing literature. We also discuss limitations of the current literature; particularly with respect to the gaps that we address as our contribution to the field in this document.

Chapter 3 introduces the substitution problem and utilizes a decision tree approach to model a problem that is limited in scope to a small number of machines, assumes short lead times, and extends over a finite time horizon. This problem serves as an introduction for the reader to concepts that we address in depth in later chapters and also is used to probe for those relationships most fruitful for further study in the more complicated modeling that follows.

In Chapter 4 we present the results of a Markov Chain analysis for the unilateral substitution problem. This problem is extensive in scope and the Markov chain approach allows us to pursue true steady state solutions for this class of problem. We implement the Markov Chain model without relying upon Poisson or exponential distribution assumptions and model with dissimilar reliability and probabilistic substitution.

In Chapter 5 we present our general substitution model with probabilistic substitution for a realistically large case and utilize a Poisson model to project true steady state results for our analysis.

In this work the primary areas that we research relate to substitution of components in the service part supply chain. We seek to introduce three enhancements to the literature. First, we include in our unidirectional model the examination of substitution using parts having dissimilar reliability from the part for which they are the substitute. Second, in all of our research, we consider probabilistic substitution; that is, we consider partial substitution policies in addition to policies that always substitute, and policies that never substitute. Finally, our general substitution model considers the case where a part is stocked exclusively for the purpose of substitution. Past work has focused on unidirectional or bidirectional substitution of one primary part for another primary part or has focused on translateral shipment of equivalent parts for the purpose of meeting inventory shortfalls. We feel that our general model captures the dynamics of a strategy that is commonly employed in practice, typically in an unplanned, ad hoc fashion.

We believe that those interested in service parts supply chains and substitution should find the following work presents some new methods of approaching these problems. We also believe that the service parts supply chain researcher, as well as those interested in substitution as a supply chain tool, will find a number of avenues that we explore to be a fruitful starting point for further exploration of these fascinating topics.

CHAPTER 2

REVIEW OF THE LITERATURE

Service parts production and inventory management has received far less research than general production and inventory problems in the literature. There are a number of key issues involved in handling service parts that make determination of inventory and production policy different, and more challenging, than the determination of a policy for conventional inventory or production. These challenges include an often very high service rate requirement due to the fact that parts failures can cause massive loss of production. Other problems encountered in the service parts supply chain, including low demand or lumpy demand (demand that occurs in clusters rather than dispersed evenly overtime), violate important assumptions of standard statistical models for economic ordering quantities and preclude the use of those well tested methods. In addition, there is frequently great uncertainty about demand, as well as a changing profile of demand, that render steady state models inapplicable.

One of the key decisions in service part management is allocating limited stock when there are not enough parts to meet all service requirements. A significant portion of service part research has been aimed at addressing the conflict that arises when, due to limited stock and high demand uncertainty, management must choose not to satisfy some demand immediately. How do we

choose what demand to satisfy when there is a high fill rate requirement associated with multiple customers and inadequate ability to meet that demand? One solution to this problem is rationing through a classification system. Veinott (1965) was the first to publish on the problem of rationing stock to several demand classes in inventory systems. He studied a periodic review inventory model with N classes of demand, zero lead time and limited ordering. Veinott's model implemented a policy of stock deployment discrimination at a critical level. The critical level policy is implemented as follows. Above the critical level, inventory is deployed as per demand. However, some level of inventory is reserved for critical demand and only that demand which is categorized as critical is satisfied when inventory is at, or below, the critical level. A demand may be classified as critical if the component failure would result in the lack of use of a piece of equipment or would harm the usage of an important piece of equipment. Similarly, demand may be tagged as critical if it is demand for a very important customer.

Nahmias and Demmy (1981) were among the first to consider multiple demand classes for continuous-review inventory models. They analyzed a (Q, r) inventory model having a critical stock level policy for a case with two classes of demand, Poisson demand arrivals, backordering, and constant lead time. Nahmias and Demmy (1981) made the rather limiting and important assumption that there could be a maximum of one outstanding order at any time.

A common problem that occurs with service parts is that demand for a given part is often very low, unpredictable, or lumpy. The problems associated

with very low, unpredictable, or lumpy demand have also been addressed through rationing. For example, Dekker et al. (1998) discuss inventory control of infrequently needed spare parts; this work included a model for critical stock level policy for a case with two classes of demand, Poisson demand arrivals, backordering, and deterministic lead time. This model does not make the assumption of at most one outstanding order. The authors derive service levels for both classes as the probability of no stock out.

Another feature that distinguishes service parts management models from standard inventory and production models is the fact that lead times are often extraordinarily short while fill rate requirements are, at the same time, extraordinarily high. Of course, there are multiple types of service, and not all service is an emergency; as a result there may be great variation in the lead time for any given demand. The earliest work in this area was accomplished by Simpson (1958) who introduced the concept of demand lead times (DLT) for base stock, multi-stage production systems. He used the term "service time" to describe inventory distribution systems where demand may not require immediate delivery, thus allowing a fixed delay. A primary observation by Simpson (1958) in this work is that demand lead time has an effect upon optimal policy opposite to the effect of supply lead time; that is, DLT reduces the required inventory to achieve the target service level.

Kocaga and Sen (2007) extend the research into rationing by combining it with the consideration of demand lead times. Their work considers a continuous-time, single-item, lot-for-lot, model with backordering under the simplifying

assumptions that there is a single item in consideration and critical levels are time invariant.

The next work that we consider advances upon earlier research, and considers new data observed each time a failure occurs as well as historical demand information. Aronis et al. (2004) put forth a model and case study results for a Bayesian approach to forecasting the demand for spare parts. The method presented in the research by Aronis et al. (2004) assumes that the failure data originate from a stationary process - that is, the model does not account for demand change resulting from changes in the number of units installed in the field. By considering the new data observed each time a failure occurs, as well as historical demand information, this model is designed to more accurately forecast the demand for spare parts,. This modification of the demand function using more current failure information is more sophisticated than the static models presented earlier.

Moore's (1971) work, "Forecasting and scheduling for past-model replacement parts" is precursor to much of the published research on service parts. Moore points out that as time passes the cost of maintaining service parts for obsolete equipment becomes increasingly burdensome. Since EOQ models assume a steady state they will consistently overshoot the demand for obsolete service parts and generate excessive inventory. Moore's work focuses on an all time requirement for service parts. An all time requirement is the total demand for the part from the point of the forecast throughout the remaining service period (RSP). The all time requirement for a part through the remaining service period

is an upper limit for the summation of all remaining demand throughout time for the service part and so the all time requirement is also referred to as the all time demand. If the all time demand can be accurately forecast, then a firm might choose to make an all time production run and gain a number of benefits from removing the obsolete part, and the overhead needed to produce it, from operation. Moore (1971) shows that by transforming sales data from an arithmetic scale to a logarithmic scale it becomes apparent that three families of curves—the parabola, ellipse and straight line are common to 85% of the parts considered. Moore's data is shown in Table 2.1 which is taken from page B208 of the above referenced work.

Table 1.1 Moore's Selection of Best Forecasting Curve

Year	Actual Sales	Forecasted Sales		
		Ellipse ¹	Parabola ²	Linear ³
1960	3,744	3,744	3,744	3,744
1961	2,945	3,036	2,531	2,618
1962	2,369	2,188	1,400	1,830
1963	1,489	1,562	782	1,280
1964	901	1,124	453	895
1965	951	816	273	625
1966	651	598	171	437
1967	427	440	110	306
1968	328	328	73	214
1969	—	242	49	149
1970	—	180	34	104
1971	—	133	24	73
1972	—	99	17	51
1973	—	73	12	35
1974	—	53	9	25
1975	—	38	7	17
1976	—	27	5	12
1977	—	18	4	8
1978	—	12	3	5
1979	—	8	2	4
1980	—	4	1	2
1981	—	1	—	1
Forecasted all-time requirements (t = 1960)		14,722	9,704	12,435
Weighted Absolute Deviation		3,435	21,062	8,585
Mean Weighted Absolute Deviation		.0055*	.0339	.0138

¹ Ellipse Equation: $\lambda^2/1.34 + \hat{t}^2/3.57 = 1$.

² Parabola Equation: $\hat{t} = 3.57 - 1.88\lambda^2$.

³ Linear Equation: $\hat{t} = -1.49\lambda + 3.57$.

* Ellipse judged to be the best fitting curve.

Moore (1971) then puts forth a dynamic inventory model to be compared against existing production schedules and adjust production of the part as needed, including the decision to make an all time run of production if warranted, so that the part may be removed from further production consideration.

Fortuin (1980) expands upon the work of Moore (1971) and derives formulae for the availability (service level), shortage risk, and obsolescence risk for service parts in the residual service period. In modeling demand for service parts, the expected demand for a service part in a given year was determined by multiplying a regression factor by the demand in the prior year. Subsequently, Fortuin (1981) presents a model to introduce significant saving through the utilization of a fictitious reduction of the remaining service based upon acceptance of a reduction in the service level near the end of the RSP. Under this method, the all-time requirement is calculated for a number of years less than the true RSP, resulting in a considerable reduction in investment in stock. The derived formulae demonstrate a 26% reduction in the all time requirement for the case where the RSP is fictitiously reduced from 10 years to 4 years. In this case, availability remained above 75% at the end of the RSP.

Cohen and Barnhart (2006) investigated the high-cost, low-demand stocking problem. The authors discuss how the decision of how to stock repair parts which are both high cost and low demand becomes extremely computationally challenging when the scope is expanded to consider what happens when warehouse capacity constraints are added. The authors demonstrate that a basic modeling approach to this new problem is very difficult,

or cannot be analytically solved in polynomial time, for many realistic sized instances of the problem. Cohen and Barnhart (2006) then present a composite-variable model and demonstrate how it improves tractability significantly. By grouping common constraints as one variable the authors demonstrate that the ability of heuristics to find solutions to the problems is greatly improved.

Fortuin and Martin (2000) set forth a number of the problems encountered when considering service part stocking strategy. Chief amongst these problems is the fact that conventional inventory theory breaks down due to a number of factors including: slow moving parts, lack of demand history, dependence upon localized conditions, and a short product life cycle. Fortuin and Martin (2006) draw the distinction between repairable parts and non-repairable parts and concentrate upon the latter. The authors also distinguish among three key life stages in the life cycle of the product. The first phase is the initial phase where very little is known about the reliability of components. The second phase is the normal phase where demand patterns are still scarce, but some information may be known - especially for fast moving parts. Finally there is the final phase where it may be necessary to place an all time order. Distinction is made between three utilizations of service parts: (1) technical systems under client control, (2) technical systems sold to customers, installed at the customer's site for the purpose of providing products or services, and (3) end products being used by customers. The authors explore two methods for removing some of the problems caused by the lumpy demand for service parts: (1) increasing demand and (2) reducing criticality. By increasing demand better forecasting can be

accomplished for the parts. As was mentioned in the work by Simpson (1958), reducing criticality can increase the demand lead time and decrease the inventory required for a given service level.

Botter and Fortuin (1999) point out many of the same difficulties noted elsewhere in the literature with respect to determining service part stocking policy including: the lumpy nature of demand, poor demand data, and high service level requirements. In this work, the researchers present a model to handle service parts inventory through the use of varying levels of criticality and the calculation of logistical parameters for the entire criticality classification.

Wong et.al (2007) analyze a two-echelon, multi-item, spare parts system with supply flexibility through lateral transshipments, and emergency direct deliveries, from the central warehouse or factory in the event of a stock-out. The authors develop a heuristic to determine the optimal stock level at each satellite warehouse. The model is structured as a combinatorial problem and is solved with a local search optimization method involving a greedy algorithm. The authors compare a single-echelon system with a two echelon system and determine that the two-echelon system is only advantageous when lateral transshipments are not permitted.

In our research we study the utilization of substitution as a mechanism for optimizing policy for service parts and consumables. One of the earliest considerations of substitution in inventory was put forth by Wagner and Whiten (1958) wherein they model an optimal inventory policy for the steady state demand case, and also for a multi-period problem, when demand is known but

varies from period to period. During the discussion the researchers suggest that the model they put forth for filling demand in period $k+t$ with inventory acquired in period k suggests that the model could be extended to the case where demand for a lesser product could be filled by prior inventory of a superior product. In this case, steel beams of superior quality could be substituted for those of lesser quality.

One of the earliest works in which substitution was the focus of the research was limited to a single-period model as longer term models become substantially more complex. This early work on substitution in a multiproduct inventory with stochastic demand was accomplished by Ignall and Veinott (1969). The authors put forth the results of their research in which they considered product substitution with proportional ordering costs and stochastic demand under a myopic ordering policy; that is, under a policy which considers minimization of only the current period costs. In this analysis the authors restrict the model to one in which total order quantity does not change. If the quantity ordered of one product increases, the quantity of other products ordered decreases by the amount so that the total order size remains the same. This work was important in that the myopic ordering policy would be adopted as a key modeling feature in many future works by other researchers.

McGillivray and Silver (1978) considered single-period, multidirectional, two-product substitution. In their case McGillivray and Silver (1978) assume that the items are essentially similar with identical variable costs and shortage penalties. The authors assumed that the substitution per replenishment cycle

would be small relative to overall inventory. The authors concluded that, even for this simple case, the savings could be significant if ordering policy for the two items together was considered as opposed to treating each item independently.

In another important single-period substitution work the single-period, multi-product, inventory problem with one-way substitution and zero setup costs was visited by Bassok et al.(1999), wherein it was demonstrated that myopic base-stock policies are optimal under the assumption that the unit substitution cost is identical among the products considered.

In production environments it is possible that one input can yield multiple outputs. An important work in this area is that of Hsu and Bassok (1999) in which the researchers consider a single-period problem with one input that yields a random number of products. They consider single-period optimization where there is the possibility of full downward substitution of products and demonstrate how to devise an efficient algorithm from the network structure of the problem.

Rao et al. (2004) consider one-way downward (higher quality to lower quality) product substitution for a multi-product inventory problem with stochastic demand and production setup costs. This treatment was limited to a single-period. These researchers develop a heuristic solution to predict optimal setup, production, and inventory levels for the single-period case. Extension of this model to a multi-period case would require relaxation of restrictions of the model resulting from the fact that levels are set at the start of the program and fixed throughout the processing of the algorithm.

Axsater (2003) put forth an approximation model for the multi-location inventory problem with unidirectional lateral transshipments. Axsater's model assumes that a base-stock policy is used at all locations. He also assumes that demand lead time is normally distributed and assumes a uniform distribution of the stationary inventory position without substitution. He shows that the single-warehouse model with unidirectional substitution can be modeled by treating products with varying quality as independent warehouses with lateral transshipment from higher quality to lower quality. However, when the model is compared against a simulation assuming Poisson demand and constant lead time, the approximation errors are not insignificant.

Liu and Lee (2006) consider one-way multiproduct substitution in the downward direction. This model is the most significant influence upon our research; particularly with respect to the work in Chapter 5. While the earlier approaches were limited to substitution upon demand arrivals, Liu and Lee (2006) extend the analysis to consider substitution upon supply arrivals. The authors develop a model and use a decomposition technique to reduce computational load approximate performance for the case of two-product and three-product substitution scenarios.

One area we will explore during this analysis is the effect of current service part and consumable part substitution decisions upon future demand. Care must be taken in that these decisions will change the expected lifetime of the part. This may result in modified ordering patterns when the customer realizes the lifetime has changed. Until the customer adapts to the modified life

cycle ordering decisions may tend to overstate or understate the true demand for parts. The earliest literature on this phenomenon was accomplished by Forrester (1958) in which he noted that information, sent in the form of orders up the supply chain, could distort variability in orders such that there was high variability in ordering even though there was not as significant or even very small variation in actual demand.

Sterman (1989) analyzes the results of “The Beer Game”, a simulation experiment in which the phenomenon now referred to as the Bullwhip Effect is manifested. Sterman (1989) characterizes the causes and potential solutions for the increased order variability. He determines that individual decisions based upon misperception of signals produces compound effects that systematically drive performance away from optimality. He presents an anchoring and adjustment heuristic for managing stock and demonstrates that the rule predicts the subjects’ behavior well. Sterman (1989) identifies a number of feedback breakdowns that account for the increasingly poor performance of the players’ decision making. The most important of these breakdowns is the failure of decision makers to note the effect their decisions have upon the overall environment.

Lee et al. (1997) cite examples from Proctor and Gamble and Hewlett-Packard and proceed to name and characterize this phenomenon as the “Bullwhip Effect”. Four main causes of the bullwhip effect are (1) demand forecast updating, (2) order batching, (3) rationing and shortage gaming, and (4) price variations. The authors discuss the role each of these causes has in

causing confusion within the supply chain and how this phenomenon causes extreme variation in the supply chain.

Contribution to the Field of Literature of this Research

The literature on service parts is not very extensive and has typically focused upon policy making with respect to rationing. The literature with respect to substitution has not focused upon service parts; the limited work involving substitution for service parts have been single-period, one-direction models. When considering warehouse substitution, the existing literature exhibits a clear hole in that it does not consider that the substituted part may have a different reliability or service life than the component for which it is substituting. This research helps fill that gap by considering the variation in reliability and service life between the primary component and the substitute in all of the models.

Another clear hole in the existing service parts supply chain literature is that there has not been consideration given to the maintenance and utilization of components whose primary purpose is to serve as a general purpose substitute. We model substitution in service and consumable part supply chains that utilize the substitution of a component that is not part of the primary product bill of materials in order to increase fill rate, reduce costs, and smooth demand.

This work also advances the literature since we model probabilistic substitution instead of using an all-or-nothing approach. To the best of our knowledge this has not been done in this area of research to date.

Although we believe that this research will contribute to the general supply chain and to the substitution literature, we feel that it will be particularly useful to the service parts sector as it will focus upon the particular needs and constraints of that segment.

CHAPTER 3

DECISION TREE ANALYSIS OF UNIDIRECTIONAL SUBSTITUTION

Introduction

One of the base criteria of this research is that the model will consider changing reliability when substitutions are made. The resulting models are very computationally complex in their full implementation. So, before we embark upon that journey, we will step back and model the problem using a decision tree approach for policy making. This is a common approach in Markovian modeling and we feel that it will serve us particularly well to begin with that process as it will help to direct the focus in later sections.

The model that we are examining assumes that the reliability or useful life of the substitute is not the same as the primary part. We feel this is a very realistic extension of the existing literature as we generally could expect that a substitute may perform better than the primary part, or it may perform worse than the primary part; however, a substitute clearly does not have the same specifications and is unlikely to have the same lifetime as a primary part. Failure to recognize this can lead to serious error in predicting the cost of substituting and planning the supply chain to minimize interruptions.

Unidirectional Substitution Model

We consider the case where there are two machine types. Machine type 1 uses Part 1 as a primary component, but machine type 1 may also use Part 2 as a substitute for Part 1. Machine type 2 uses Part 2 as its primary part with no allowable substitute.

Nomenclature of Decision Tree Unidirectional Substitution Model

C_1 = Unit cost of Part 1

C_2 = Unit cost of Part 2

S = Fixed ordering cost for total order of all parts 1 and parts 2 per order

h^1 = Holding cost of Part 1 per time period

h^2 = Holding cost of Part 2 per time period

b_1 = Cost of not operating machine type 1 during a given time period

b_2 = Cost of not operating machine type 2 during a given time period

I_t^1 = Inventory level of Part 1 at the beginning of period t

I_t^2 = Inventory level of Part 2 at the beginning of period t

T = Number of time periods

x = Number of machines using Part 1 as a primary part

y = Number of machines using Part 2 as a primary part

x_1 = Number of Part 1 installed on machine type 1 during time period t

x_2 = Number of Part 2 installed on machine type 1 during time period t

x_3 = Number of machine type 1 idle during time period t

y_1 = Number of Part 2 installed on machine type 2

y_2 = Number of machine type 2 idle during time period t

p_1^1 = Probability that a Part 1 installed in machine type 1 will fail during period 1

p_1^2 = Probability that a Part 2 installed in machine type 1 will fail during period 1

p_2^2 = Probability that a Part 2 installed in machine type 2 will fail during period 2

$F_{1,t}^1$ = Number of failures of Part 1 on machine type 1 during time period t

$F_{1,t}^2$ = Number of failures of Part 2 on machine type 1 during time period t

$F_{2,t}^2$ = Number of failures of Part 2 on machine type 2 during time period t

Problem Description for Decision Tree Approach

We consider the case with two machine classes, machine type 1 and machine type 2. Part 1 works in machine type 1. Part 2 works in machine type 2 as a primary part and works in machine type 1 as a substitute part. The possible flow of parts is as shown in Figure 3.1. The model assumes that demand is always met with a primary part if that primary part is in stock. In the event that substitution of one component for another occurs we assume that the substitution only occurs after all demand for the substitute product's primary use has been fulfilled.

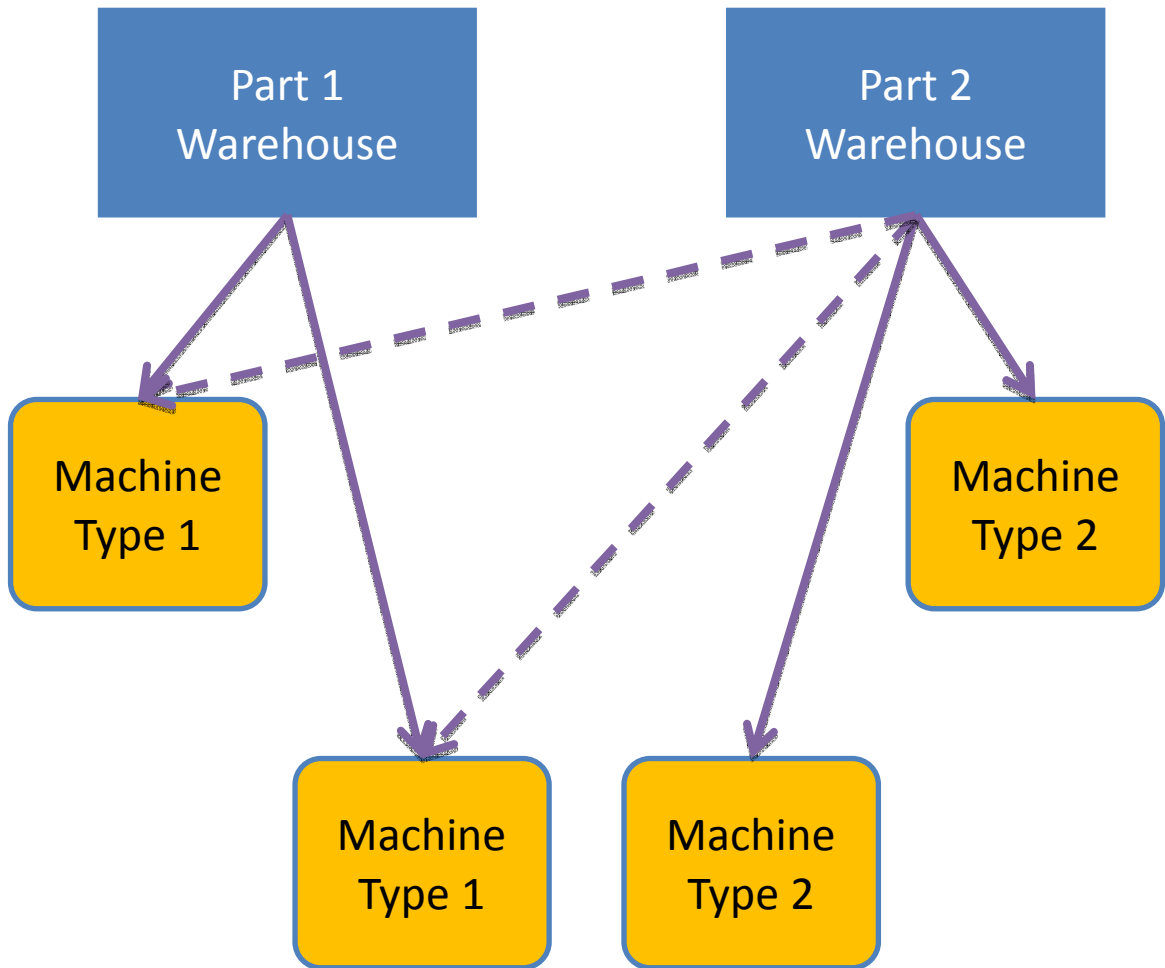


Figure 3.1 Simplified Flow of Parts Through the System

The model assumes that the cost for a part is realized when that part is removed from inventory to be placed into service. This is a common method for allocating cost that allows us to effectively recognize the cost of part substitution. Movement from one state to another is determined by the priority rules where any down machine is brought up by the correct spare part for its machine type if that part is in inventory. If both machine type 1 and machine type 2 require a part, and there is Part 2 in stock but no Part 1 in stock, then machine type 2 is

given priority for Part 2. Once a substitution has occurred the substitute will be left in place until it fails.

Transformation of State Rules for the Decision Tree Model

In the decision tree model for the unidirectional substitution problem the movement from the state at time t to the state at time $t+1$ is governed by the systematic application of rules for transformation upon the arrival of demand for parts and the arrival of parts. It is important to note that these are discrete event operations that happen upon the occurrence of any of the key events: which include part failure, or part arrival, for either part. These rules are implemented during the algorithm by following the transformation guidelines in Table 3.1 and result in the transformation as shown in Table 3.2.

Table 3.1 State Transformation Rules

Rule Number	Test	Transformation Rule
1	If $I_t^1 \geq (F_1^1 + F_1^2 + x_3)$ and $I_t^2 \geq (F_2^2 + y_2)$	$\{I_t^1 - (F_{1,t}^1 + F_{1,t}^2 + x_3), (I_t^2 - (F_{2,t}^2 + y_2)), (x_1 + F_{1,t}^1 + x_3), (x_2 - F_{1,t}^2), 0, (y_1 + y_2), 0\}$
2	If $I_t^1 \geq (F_{1,t}^1 + F_{1,t}^2 + x_3)$ and $I_t^2 \leq (F_{2,t}^2 + y_2)$	$\{I_t^1 - (F_{1,t}^1 + F_{1,t}^2 + x_3), 0, (x_1 + F_{1,t}^2 + x_3), (x_2 - F_{1,t}^2), 0, (y_1 + (I_t^2 - F_{2,t}^2)), (y_2 - (I_t^2 - F_{2,t}^2))\}$
3	If $I_t^1 \leq (F_{1,t}^1 + F_{1,t}^2 + x_3)$ and $I_t^2 \geq (F_{1,t}^1 + F_{1,t}^2 + x_3 + F_{2,t}^2 + y_2)$	$\{0, (I_t^2 - (F_{1,t}^1 + F_{1,t}^2 + F_{2,t}^2 + x_3 - y_2 - I_t^1)), (x_1 + (I_t^1 - F_{1,t}^1)), (x_2 + ((F_{1,t}^1 + x_3) - I_t^1)), 0, (y_1 + y_2), 0\}$
4	If $I_t^1 < (F_1^1 + F_1^2 + x_3)$ and $(F_2^2 + y_2) \leq I_t^2 < (F_1^1 + F_1^2 + x_3 + F_2^2 + y_2)$	$\{0, 0, (x_1 + (I_t^1 - F_{1,t}^1)), (x_2 + I_t^2 - y_2 - F_{1,t}^2), (x_3 - ((I_t^1 + I_t^2 - F_{2,t}^2 - y_2) - (F_{1,t}^1 + F_{1,t}^2))), (y_1 + y_2), 0\}$
5	If $I_t^1 < (F_{1,t}^1 + F_{1,t}^2 + x_3)$ and $I_t^2 < (F_{2,t}^2 + y_2)$	$\{0, 0, (x_1 + (I_t^1 - F_{1,t}^1)), (x_2 - F_{1,t}^2), (x_3 - (I_t^1 - (F_{1,t}^1 + F_{1,t}^2))), (y_1 + (I_t^2 - F_{2,t}^2)), (y_2 - (I_t^2 - F_{2,t}^2))\}$

Table 3.2 State Assignment After Movement From Old State to New State

Part 1 demand results in the following	
When the Following Occurs	Current State Variables Transform To This State
Failure of a Part 1 in machine type 1 with Part 1 in inventory	$(I_t^1 - 1), I_t^2, x_1, x_2, x_3, y_1, y_2$
Failure of a Part 1 in machine type 1 with no Part 1 in inventory but Part 2 in inventory and substitution is approved.	$I_t^1, (I_t^2 - 1), (x_1 - 1), (x_2 + 1), x_3, y_1, y_2$
Failure of a Part 2 in machine type 1 with Part 1 in inventory	$(I_t^1 - 1), I_t^2, (x_1 + 1), (x_2 - 1), x_3, y_1, y_2$
Failure of a Part 1 in machine type 1 with neither Part 1 in inventory nor Part 2 in inventory or with Part 2 in inventory and substitution is not permitted.	$(I_t^1 - 1), I_t^2, (x_1 - 1), x_2, (x_3 + 1), y_1, y_2$
Failure of a Part 2 in machine type 1 with neither Part 1 in inventory nor Part 2 in inventory or with Part 2 in inventory and substitution is not permitted	$(I_t^1 - 1), I_t^2, x_1, (x_2 - 1), (x_3 + 1), y_1, y_2$
Failure of a Part 2 in machine type 1 with no Part 1 in inventory but with Part 2 in inventory and substitution is approved.	$I_t^1, (I_t^2 - 1), x_1, x_2, x_3, y_1, y_2$
Part 2 demand results in the following	
When the Following Occurs	Current State Variables Transform To This State
Failure of a Part 2 in machine type 2 with Part 2 in inventory	$I_t^1, (I_t^2 - 1), x_1, x_2, x_3, y_1, y_2$
Failure of a Part 2 in machine type 2 with no Part 2 in inventory	$I_t^1, (I_t^2 - 1), x_1, x_2, x_3, (y_1 - 1), (y_2 + 1)$
Part Arrival results in the following	
Inventory of Part 1 arrives	$(I_t^1 + (S_1 - s_1)), I_t^2, x_1, x_2, x_3, y_1, y_2$
Inventory of Part 2 arrives	$I_t^1, (I_t^2 + (S_2 - s_2)), x_1, x_2, x_3, y_1, y_2$
Replenishment of stock after Part 1 inventory arrivals results in the following	
<i>Iff</i> $x_3 > 0$	$(I_t^1 - 1), I_t^2, (x_1 + 1), x_2, (x_3 - 1), y_1, y_2$
Replenishment of stock after Part 2 inventory arrivals results in the following	
<i>Iff</i> $x_3 > 0$ and $I_t^1 \leq 0$	$(I_t^1 - 1), I_t^2, x_1, (x_2 + 1), (x_3 - 1), y_1, y_2$
<i>Iff</i> $y_2 > 0$	$I_t^1, (I_t^2 - 1), x_1, x_2, x_3, (y_1 + 1), (y_2 - 1)$

Where

- If $I_t^1 > 0$ then $x_3 = 0$
- If $I_t^2 > 0$ then $x_3 = 0$
- If $I_t^2 > 0$ then $y_2 = 0$
- If $I_t^2 < 0$ then $y_2 = 0$
- If $I_t^1 < 0$ and I_t^2 then $x_3 = 0$

Decision Tree Model Results

In order to find a solution to this problem for a simple case it was determined to model the unidirectional substitution problem as a decision tree in order to examine the effects of various parameters on the expected value of supply chain costs. The decision tree modeling allows the analysis of scenarios that are not steady state and extend beyond a single time period - two key enhancements of prior models. The tree allows the exploration of the possibility that the service life for a substitute part may not be the same as its life on its primary application, or the same as the life of the primary component on the machine upon which it is substituting. In addition, we can probe for longer term effects that cascade in subsequent time periods due to the substitution of the part with a dissimilar reliability. Also, the decision tree model will allow us to examine the effects of probabilistic substitution. This section will serve as a foundation for later modeling.

In this model it is assumed that there are two part types. Part 1 can only be used for application on machine type 1. Part 2 can be used for its primary application on machine type 2 but it can also be used as a substitute for Part 1 on machine type 1.

It is assumed that for the span of the decision tree model that no replacements can be received. This enables investigation of the realistic case where the only alternative to downtime is substitution. In this model we have one machine type 1 running Part 1 and one machine type 2 running Part 2

The analysis begins with an initial inventory of two units of Part 1 and two units of Part 2. We assume that the units are inspected at fixed intervals and replacements are made as required if possible. Holding costs are applied at each inspection cycle. Backorder costs are also applied at each inspection cycle. The backorder costs may reflect loss of customer goodwill, environmental penalties, or the costs of lost production.

If a machine is idle the backorder charges are accrued at each inspection cycle. The state of the system is described using the tag convention: $\{I_t^1, I_t^2, x_1, x_2, x_3, y_1, y_2\}$. For example, in the following analysis the system begins in the ground state $\{2,2,1,0,0,1,0\}$. This signifies that there are two Part 1 in inventory, two Part 2 in inventory, one machine type 1 running Part 1, no machine type 1 running Part 2, no machine type 1 idle, one machine type 2 running Part 2, and no machine type 2 idle.

In this analysis, starting from any given state, there are four possible outcomes. The first possibility is that there is no change in the system state. The second possibility is a part on machine type 1 fails. The third possibility is that a part on machine type 2 fails. Finally, it is possible that a part fails on both machine type 1 and on machine type 2. In order to prepare the algorithm it was

necessary to enumerate all possible states of the system. The enumeration is shown in Table 3.3.

Table 3.3 State Transformation Mapping for Decision Tree When Substitution is Allowed

Old State (and New State if No Failures)	New State if Part in Machine Type 1 Fails But Part in Machine Type 2 Does Not Fail	New State if Part in Machine Type 2 Fails But Part in Machine Type 1 Does Not Fail	New State if Part in Machine Type 1 Fails and Part in Machine Type 2 Fails
2,2,1,0,0,1,0	1,2,1,0,0,1,0	2,1,1,0,0,1,0	1,1,1,0,0,1,0
2,1,1,0,0,1,0	1,1,1,0,0,1,0	2,0,1,0,0,1,0	1,0,1,0,0,1,0
2,0,1,0,0,1,0	1,0,1,0,0,1,0	2,0,1,0,0,0,1	1,0,1,0,0,0,1
2,0,1,0,0,0,1	1,0,1,0,0,0,1	Cannot happen	Cannot happen
1,2,1,0,0,1,0	0,2,1,0,0,1,0	1,1,1,0,0,1,0	0,1,1,0,0,1,0
1,1,1,0,0,1,0	0,1,1,0,0,1,0	1,0,1,0,0,1,0	0,0,1,0,0,1,0
1,0,1,0,0,1,0	0,0,1,0,0,1,0	1,0,1,0,0,0,1	0,0,1,0,0,0,1
1,0,1,0,0,0,1	0,0,1,0,0,0,1	Cannot happen	Cannot happen
0,2,1,0,0,1,0	0,1,0,1,0,1,0	0,1,1,0,0,1,0	0,0,0,1,0,1,0
0,1,1,0,0,1,0	0,0,0,1,0,1,0	0,0,1,0,0,1,0	0,0,0,0,1,1,0
0,1,0,1,0,1,0	0,0,0,1,0,1,0	0,0,0,1,0,1,0	0,0,0,0,1,1,0
0,0,1,0,0,1,0	0,0,0,0,1,1,0	0,0,1,0,0,0,1	0,0,0,0,1,0,1
0,0,0,1,0,1,0	0,0,0,0,1,1,0	0,0,0,1,0,0,1	0,0,0,0,1,0,1
0,0,0,0,1,1,0	Cannot happen	0,0,0,0,1,0,1	Cannot happen
0,0,1,0,0,0,1	0,0,0,0,1,0,1	Cannot happen	Cannot happen
0,0,0,1,0,0,1	0,0,0,0,1,0,1	Cannot happen	Cannot happen
0,0,0,0,1,0,1	Cannot happen	Cannot happen	Cannot happen

Next, in order to evaluate the model, the Decision Tools Suite from Palisade software was used. The logic was entered into PrecisionTree, after which the optimal path was tracked for a number of scenarios and sensitivity

analysis performed in order to determine the answer to a number of key questions including:

- Was there significant benefit to the expected cost resulting from substitution?
- Was there significant change to expected cost if the substitute part was more reliable or less reliable than the part for which it was substituted?
- What parameters influenced the choice to substitute?
- Which parameters had significant impact upon expected cost?

In order to demonstrate the implementation of the model, a small section of the tree is shown in Figure 3.2. Note that in this branch the user is faced with a substitution choice, and based upon the results of that choice the probability of failure and potential future states are altered. Subsequent decisions depend upon whether the model proceeds along the substitute path or proceeds along the non-substitute path. Since Part 2 will remain in machine type 1, even if stock of Part 1 later becomes available, the dissimilar reliability of the substitute will have effects into the future and will impact all expected values upon that path. Policy makers must recognize that this phenomenon is occurring and plan for the modified demand in future ordering. The base case simulation values for costs and component reliability are shown in Table 3.4 and Table 3.5.

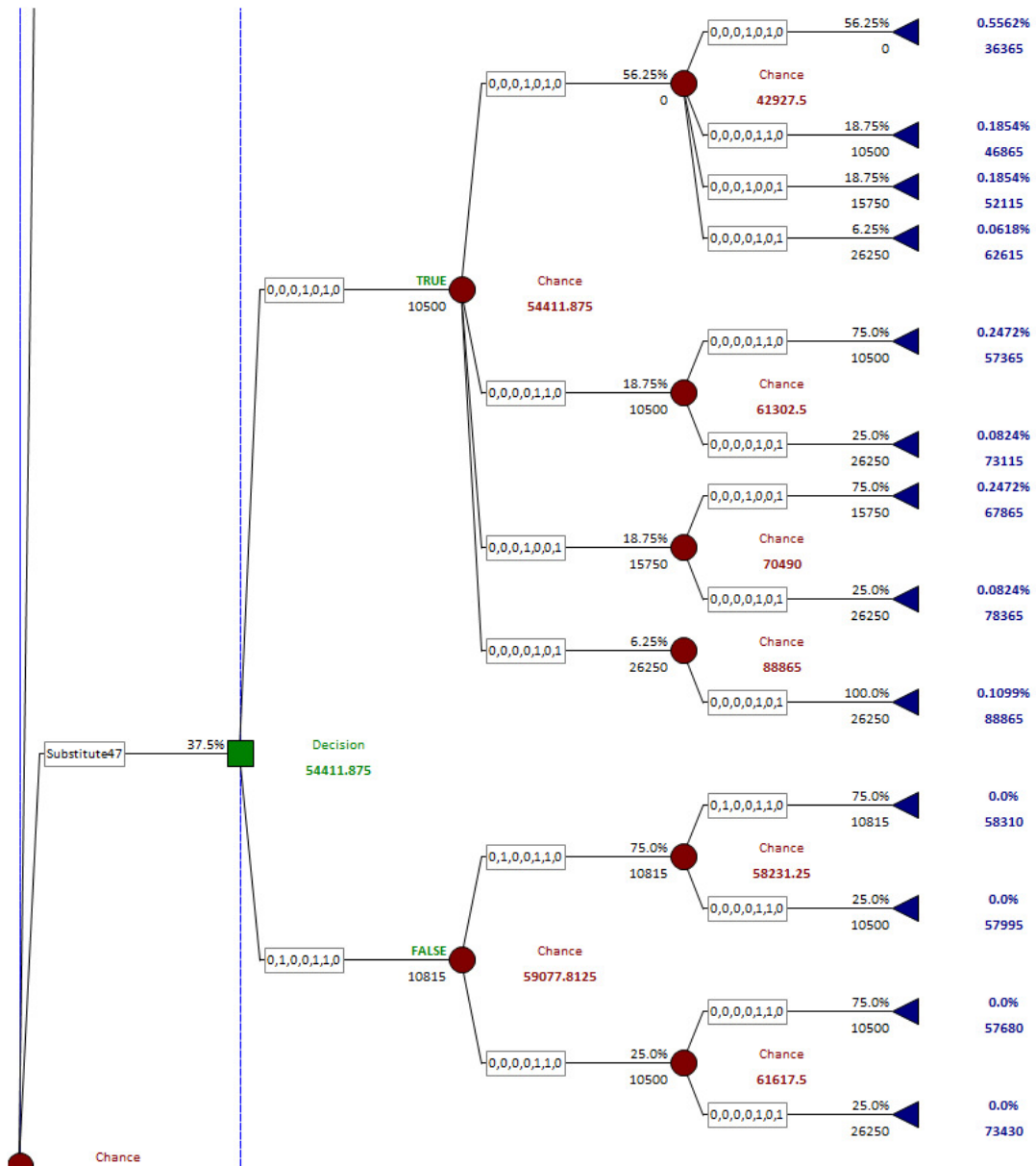


Figure 3.2 Branch of Decision Tree Showing Expected Value of Substitution Choices

Table 3.4 Base Case Costs for Decision Tree Model

Cost Component	Cost in \$
C_1	\$7,000
C_2	\$10,500
h_1	\$210
h_2	\$315
b_1	\$10,500
b_2	\$15,750

Table 3.5 Probabilities Used in Decision Tree Model

Probability	Value
p_1^1	0.5
p_2^2	0.25
p_1^2	0.25

Utilizing these parameters the financial results along the optimal path had an expected cost of \$33,324. Figure 3.3 demonstrates the cumulative probability of expected values for this scenario. Figure 3.3 shows the cumulative probability that the expected value of the “optimal” path is less than a given value.

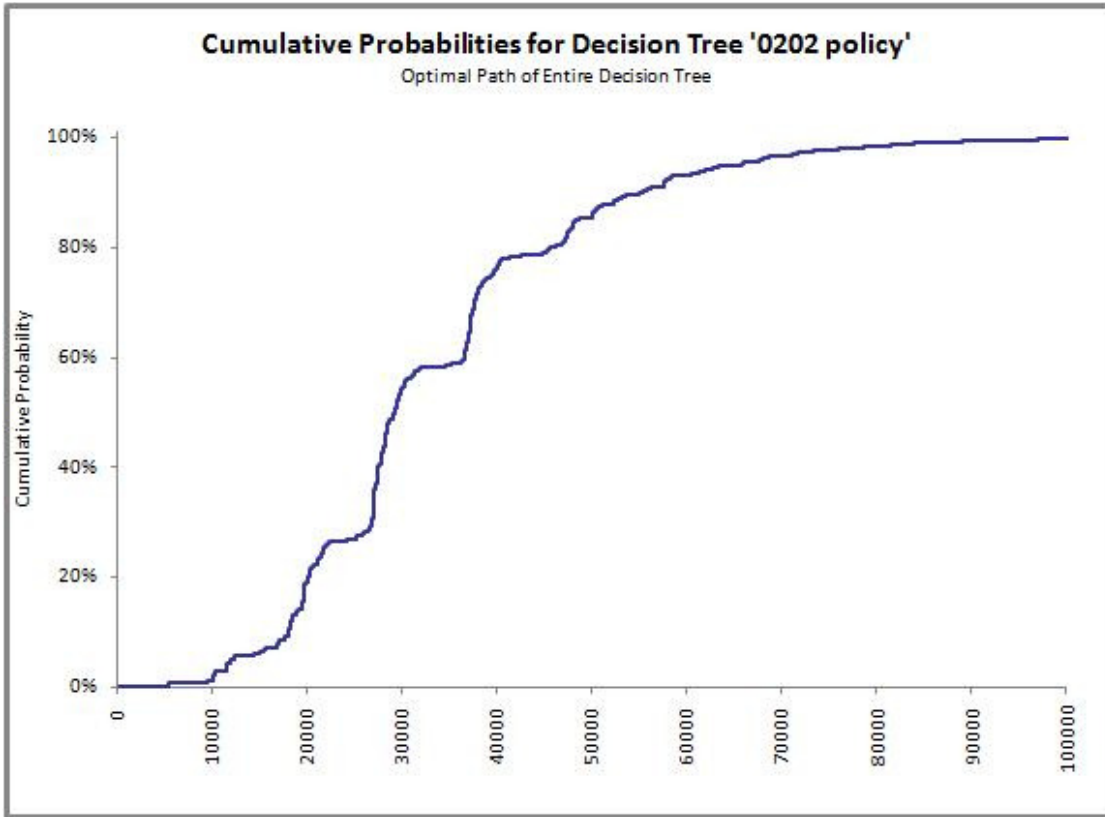


Figure 3.3: Cumulative Probability of Expected Value for Base Case Analysis

In Table 3.6 the choices made upon the optimal path are shown. It is very revealing to note that in almost every case upon the optimal path the choice to substitute was made when available. This demonstrates that the availability of substitution does have significant financial benefit. Table 3.6 illustrates the choices made at each substitution decision. The optimal choice is the node that results in the lowest expected cost. The chart gives the arrival probability for that decision node, i.e. the probability that the particular node will be traversed. The chart shows the benefits of a preferred choice in the form of the reduction to expected cost for making the optimal choice.

Table 3.6 Choices Made on Optimal Path and Resulting Benefit

Decision	Optimal Choice	Arrival Probability	Benefit of Correct Choice
'Decision' (M124)	0,1,0,1,0,1,0	0.7416%	-315
'Decision' (M134)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (M154)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (M222)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (M268)	0,1,0,1,0,1,0	0.7416%	-315
'Decision' (M278)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (M298)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (M312)	0,1,0,1,0,1,0	0.7416%	-315
'Decision' (M322)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (L328)	0,1,0,1,0,1,0	1.9775%	-315
'Decision' (M338)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (L358)	0,0,0,1,0,1,0	0.6592%	-7113.75
'Decision' (M382)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (M418)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (L444)	0,0,0,1,0,1,0	0.6592%	-7113.75
'Decision' (M522)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (M638)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (M674)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (L700)	0,0,0,1,0,1,0	0.6592%	-7113.75
'Decision' (M798)	0,1,0,1,0,1,0	0.7416%	-315
'Decision' (M808)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (M828)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (M856)	0,0,0,1,0,1,0	0.2472%	-15435
'Decision' (K924)	0,1,0,1,0,1,0	3.2734%	-14604.84375
'Decision' (M946)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (L972)	0,0,0,1,0,1,0	0.6592%	-7113.75
'Decision' (K1020)	0,1,0,0,1,1,0	1.7578%	-4370.625
'Decision' (M1036)	0,0,0,1,0,1,0	0.2472%	-315
'Decision' (L1062)	0,0,0,1,0,1,0	0.6592%	-7113.75
'Decision' (K1110)	0,0,0,1,0,1,0	1.7578%	-4665.9375

After modeling the base case scenario, sensitivity analysis was performed wherein we altered the ratio of a number of model parameters in order to measure the impact on expected value (expected cost). Figure 3.4 is a tornado

diagram that shows the impact upon expected value resulting from a 50% increase and a 50% decrease of five key parameter ratios.

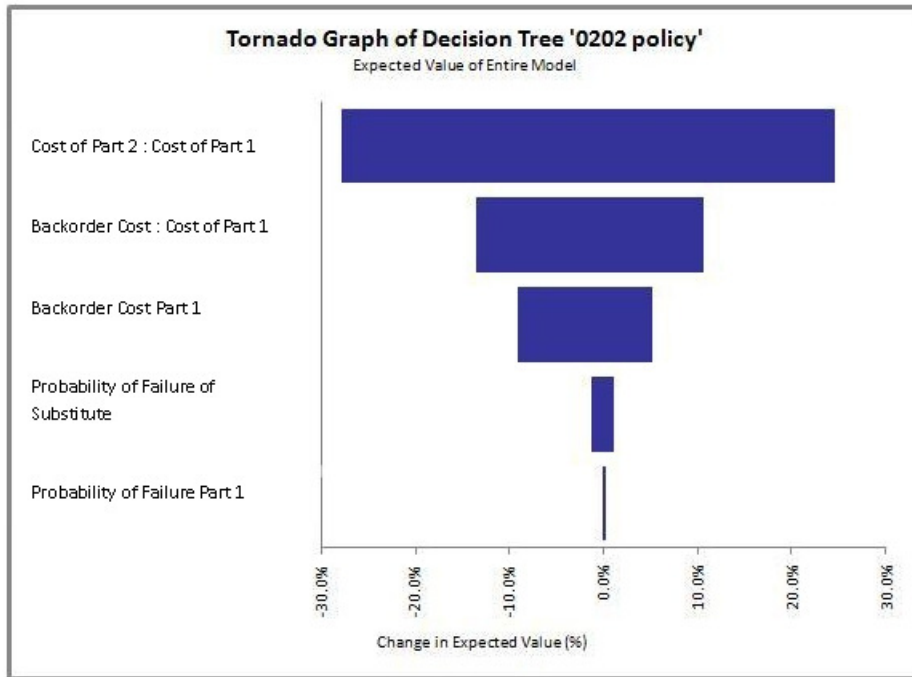


Figure 3.4 Tornado Diagram of Sensitivity Analysis

From the tornado diagram in Figure 3.4 we can clearly visualize the sensitivity of expected value to changes in key parameters. In order of decreasing importance those changes are:

- Ratio of the cost of Part 2 to the cost of Part 1
- Ratio of the backorder cost of Part 1 to the price of Part 1
- Backorder cost
- Probability of failure of Part 2 on machine type 1

The probability of failure of Part 1 on machine type 1 was not particularly significant over this short time frame. It is possible that some of this variation

was simply a linear expansion of total overall price increase affecting the cost and was not a variable that impacted the overall efficiency of the operation. In order to determine which variables had significant effect on expected value a spider diagram (Figure 3.5) was prepared to determine if a linear relationship existed. No one-to-one mapping can be seen in the spider graph so it is clear that the flexibility of the substitution model can offset price increases or efficiency drops. In addition, individual charts of changes in expected value with respect to price were prepared to examine whether changes simply represented inflation, or if the changes caused a response in the model which could teach us something about the system.

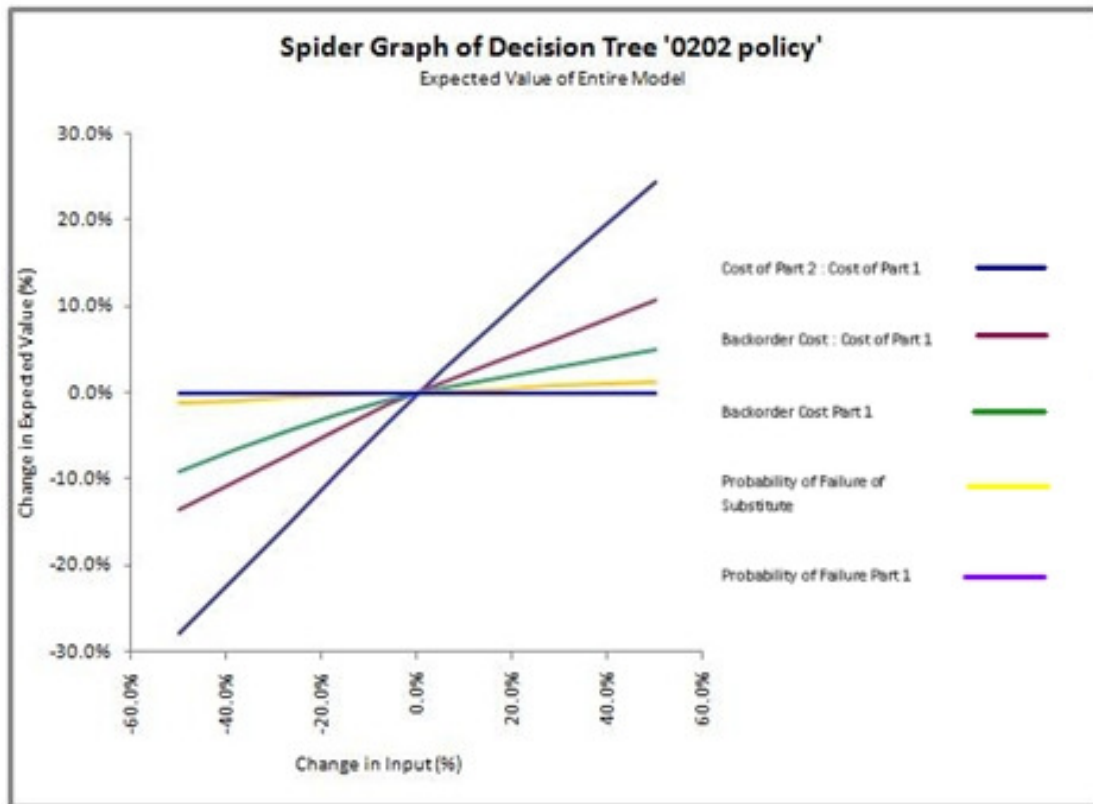


Figure 3.5 Spider Diagram of Sensitivity Measurement Changes

Figure 3.6 explores the results of changes to backorder costs on the expected value of the model. Changes in backorder cost showed a decreasing impact as the overall backorder cost became larger. This reflects improvements due to increased propensity to substitute.

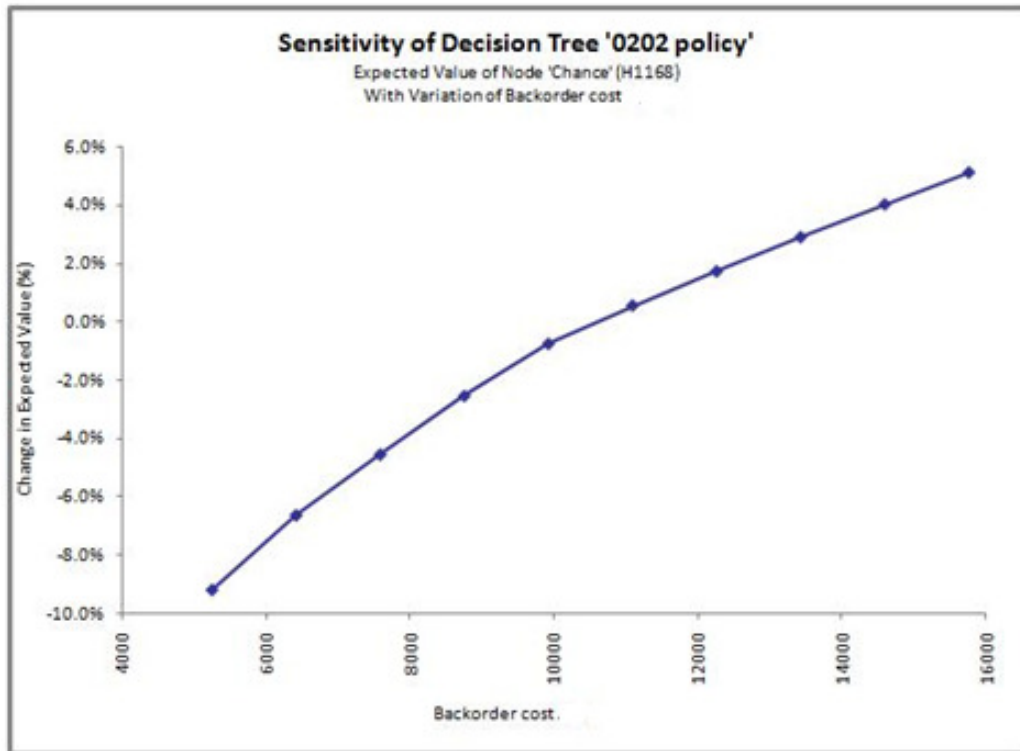


Figure 3.6 Sensitivity of Expected Value to Backorder Cost of Part 1

Further sensitivity analysis demonstrated that those models with the lowest cost path for any scenario involved some degree of substitution, and the overall lowest cost models (with optimal parameter values) involved heavy substitution. Table 3.7 shows the results of a number of these trials. In no case was a lowest cost path found that involved no substitution, even when the

benefits of substitution were reduced by low reliability of Part 2 on machine type 1 or a lowered backorder penalty.

Table 3.7 Expected Value of Various Scenarios

Canister A Price	Canister B Price	EV	Substitute	Failure Ratio	Backorder Ratio
\$7,000	\$10,500	\$30,393	Mixed	0.25	1
\$7,000	\$10,500	\$30,750	Mixed	0.5	1
\$7,000	\$10,500	\$30,071	Mixed	0.1	1
\$7,000	\$7,000	\$25,230	Always	0.25	1
\$7,000	\$8,000	\$26,733	Mixed	0.25	1
\$7,000	\$10,500	\$33,324	Always	0.25	1.5
\$7,000	\$10,500	\$31,889	Mixed	0.25	1.25

Summary of Decision Tree Model

Overall, the model worked well and produced interesting results that beg further study. The following sections will utilize a Markov Chain and a Poisson process in order to increase the time horizon so that longer term effects of substitution, and varying effectiveness of the substitute, can be analyzed. From this section, we can see that even utilizing a limited model demonstrates that substitution can have important results for expected value.

What we have accomplished in this section is to demonstrate an approach to move the study of substitution for service parts beyond a myopic model into multi-period analysis. We also see the limitations of the myopic model manifested as the cascade effect of current time period decisions upon future time period choices. In addition, we have created the approach we will model with the Markov chain to allow the consideration of probabilistic substitution and dissimilar reliability.

CHAPTER 4

UNIDIRECTIONAL SUBSTITUTION USING MARKOV CHAINS

Introduction

In this section we look at a variation of the normal inventory management policy where unidirectional substitution is employed as a policy in order to improve the customer service to cost relationship. In today's increasingly global economy, firms find themselves with significant customer installations far from the manufacturing center and primary warehouse bases. For example, a manufacturer may have its production and distribution centers in the United States but might have an important customer located in Sicily. The company may have no other customers in Sicily, but making sure that this customer is serviced properly is an important concern, because this customer is an important component of the firm's business. As a result, it is important to make sure that parts needed to service this customer's machinery are available in Sicily in order to minimize downtime.

During our research we examine policy optimization for managing service parts in satellite or remote warehouses; particularly with respect to those scenarios where inventory costs are high, and inventories must be kept small, which together present a number of challenges that defy conventional optimal inventory modeling. One of the primary assumptions for the service part policy

we are examining is that customers are highly intolerant of unscheduled downtime. When a component failure occurs it is critical to bring the system back on line immediately. This means that the demand lead time for service parts is often very short. A number of scenarios meet this condition including: parts critical to a manufacturing process, parts used in life safety systems, and parts required to maintain regulatory requirements.

An important differentiation for service part policy vs. standard production and inventory policy is the fact that the demand is often unpredictable and can be very low for a given part. In addition demand for service parts may deteriorate over time, or the target machine may no longer be in use, and these factors can result in an extreme risk of obsolescence. As a result, it is generally desirable to limit the amount of parts in storage or the size of production runs. In addition, given a low and unpredictable demand rate, it is not always feasible to interrupt normal production in order to make a small product run on very short demand lead time. Finally, as many of these service facilities are located great distances from the factory, the shipping times and customs clearance times are not insignificant and introduce a great deal of variability into supply lead times. The end result of these factors is that service part inventories are often small, and it is not safe to assume that we can rely upon the factory or suppliers to immediately produce a replacement component.

The traits we have mentioned create a scenario where substitution of one component for another component is very attractive. The use of substitution can

reduce production quantities and lead times while increasing service fill rates.

This utilization of unidirectional substitution is the focus of our research.

Nomenclature of Markov Chain Unidirectional Substitution

State Variables

i_j^1 = Inventory of Part 1 at state j

i_j^2 = Inventory of Part 2 at state j

i_j^{21} = Number of substitutes at state j

v_j^1 = Number of arrivals of Part 1 arriving 1 periods in the future at state j

w_j^1 = Number of arrivals of Part 2 arriving 1 periods in the future at state j

v_j^2 = Number of arrivals of Part 1 arriving 2 periods in the future at state j

w_j^2 = Number of arrivals of Part 2 arriving 2 periods in the future at state j

Parameters

p_t^1 = Probability that a Part 1 installed in machine type 1 will fail during period t

p_t^2 = Probability that a Part 2 installed in machine type 2 will fail during period t

p_t^{21} = Probability that a Part 2 installed in machine type 1 will fail during period t

n^1 = Number of machines using Part 1 as a primary part

n^2 = Number of machines using Part 2 as a primary part

θ = Lead time for new part arrival (assumed to be equal for Part 1 and Part 2)

Decision Variables

s^1 = Target for Part 1 stock

s^2 = Target for Part 2 stock

P^s = Probability that a substitution will occur

Algorithmic Variables

I_t^1 = Inventory level of Part 1 at the beginning of period t

I_t^2 = Inventory level of Part 2 at the beginning of period t

δ_t = Number of machine type 1 running with Part 2 as a substitute at the beginning of period t

$I_{\max t}^1$ = Maximum number of Part 1 at the beginning of period t

I_{\max}^2 = Maximum number of Part 2 at the beginning of period t

δ_{\max} = Maximum number of machine type 1 running with Part 2 as a substitute

T = number of time periods

m_t^1 = Number of failures of Part 1 on machine type 1 during period t

m_t^2 = Number of failures of Part 2 on machine type 2 during period t

m_t^{21} = Number of failures of Part 2 on machine type 1 during period t

$m_j^{1 \max}$ = Maximum number of failures of Part 1 on machine type 1 from state set j

$m_j^{2 \max}$ = Maximum number of failures of Part 2 on machine type 2 from state set j

$m_j^{21 \max}$ = Maximum number of failures of Part 2 on machine type 1 from state set

j

\mathbb{U}_j = The set of all valid and unique state sets

\mathbb{U} = The set of all valid and unique state sets

J = Number of unique states

$n_j^{21 \max}$ = Maximum number of substitutions possible from state set j

n_t^{21} = Actual number of substitutions per cycle

q_1 = Number of binomial trials Part 1 on machine type 1

q_2 = Number of binomial trials Part 2 on machine type 2

q_{21} = Number of binomial trials Part 2 on machine type 1

F_t^1 = Number of failures of Part 1 on machine type 1 during time period t

F_t^2 = Number of failures of Part 2 on machine type 2 during time period t

F_t^{21} = Number of failures of Part 2 on machine type 1 during time period t

A = Transition probability matrix

Steady State Values

π_j = Steady state probability of state j

I_{ss}^1 = Steady state inventory of Part 1

I_{ss}^2 = Steady state inventory of Part 2

Δ_{ss} = Steady state level of substitution

v_{ss} = Steady state purchases of Part 1

w_{ss} = Steady state purchases of Part 2

F_{ss}^1 = Steady state fill rate of Part 1

F_{ss}^2 = Steady state fill rate of Part 2

F_{ss}^T = Steady state total fill rate

B_{ss}^1 = Steady state backorder rate of Part 1

B_{ss}^2 = Steady state backorder rate of Part 2

Establishing the Existence of the Markov Property and Ergodicity

In this work we consider the unidirectional (one-way) substitution problem for a case in which there are two machine types. Each machine type uses a

particular part similar enough to other machine type's part that the part used on machine type 2 (Part 2) can be used as a substitute for the primary part on machine type 1 (Part 1) if necessary. However, the parts are not perfect substitutes because the standard part on machine type1 cannot be used as a substitute on machine type 2. In addition, when Part 2 is substituted for Part 1, the expected lifetime of Part 2 on machine type 1 may be different from the expected lifetime of Part 1 on machine type 1 and may also be different than the expected lifetime of Part 2 on machine type 2.

In order to utilize a Markov Chain first we must ensure that we develop the model in such a way that we maintain the Markov Property. We do this through careful state design and transformation rule definition.

The Markov Property requires that the conditional probability of subsequent states of the process, given the present state, depends only upon the present state and not past states; i.e. it is conditionally independent of these past states. Formally, $\Pr(X_n | X_{t=1}, X_{t=2}, X_{t=3} \dots X_{t=n}) = \Pr(X_n | X_n)$. Often, in order to ensure that the Markov Property is retained, researchers model the problem assuming a probability density function for which the no memory property is a characteristic feature. Examples of this sort of distribution include exponential distributions and Poisson arrivals. However, we wish to consider a case in which there is no guarantee that arrivals are exponential, or that Poisson arrivals exist, and the sample size is not large enough to assume a Poisson approximation.

Our approach to ensuring that we retain the Markov Property is very straightforward. We carefully design each state variable set such that sufficient

information is included in that state set to determine any subsequent state. In addition, we ensure that the method used to transition from the present state set to the next state set incorporates only the information in the present state set.

Our objective is to predict steady state values for inventory levels, fill rates, ordering rates, and backorder rates, for given set point and substitution policies. As such, in addition to retaining the Markov Property, the model must yield a transition probability matrix that is ergodic.

First, we set forth the sequence of operations and method of accounting that is used in this model, as these assumptions underpin the Markovian assumptions upon which this model relies. The following sequence of events is assumed to occur:

1. Each morning inventory arrives prior to the service person's audit of the system.
2. The service person notes any failures from the prior period and makes replacements in the following sequence:
 - i. Failures on machine type 1 of either Part 1 or Part 2 are repaired with Part 1 until Part 1 stock is depleted.
 - ii. Failures on machine type 2 are repaired with Part 2 stock until Part 2 stock is depleted.
 - iii. Failures on machine type 1 of either Part 1 or Part 2 are repaired with Part 2, if adequate Part 2 is available, with some probability of substitution set by policy. The probability of substitution is applied to each potential instance of substitution.

3. An inventory of Part 1 and Part 2 is taken and parts of each type are ordered so that any down machine can be repaired with the primary part for that machine class and so that inventory levels for each part will reach the target stock level after such repairs are made.
4. The state of the inventory and outstanding orders is logged and this logged value is the new state value for this time period.
5. Once a substitution has been performed the part remains in place until it fails.

In the case of the one-way substitution problem we are interested in knowing the inventory of each part and the number of machines currently running with a substitute, so each of these values is captured with a state set member. In order to predict future states, using only information captured in the current state set, we must also have variables that indicate the arrival of new parts. These variables, in conjunction with the value of current inventory and substitution levels, will yield the probability of transitioning to a new state when combined with the probability of failure and the probability of substitution. As a result, the state variable set will include state set members necessary to capture the arrival of spare parts. The collection of all necessary state set members required to uniquely identify a possible state comprise a state variable. The collection of all state variable sets describes the universe of possible states, which we refer to as U .

A state set model that incorporates all necessary information for this unidirectional substitution problem is of the form $\{I_t^1, I_t^2, \delta_t, v_1, w_1, \dots, v_\theta, w_\theta\}$. We refer

to each member variable of the set as a state variable member and refer to the entire set of such variables as a state variable set. Collectively, the universe of state variable sets describes every possible grouping of inventory, equipment operational status, and outstanding orders. After assembling an array of all unique possible state variable sets, we can utilize a Markov Chain to calculate a steady state transition matrix and steady state values for a wide variety of measures of goodness. With this state set definition, and the rules for action described above, we ensure that the Markov Property is retained. We maintain the integrity of this assumption by deriving all state sets from one seed of this class using transformation rules based upon the aforementioned operations.

Proof of Accessibility and Irreducibility

For some arbitrary but fixed states $f, g \in E$; by definition, g is accessible from f if and only if $p_{f,g}^n > 0$ for some $n \geq 0$. Let $t_g =$ the number of steps until the Markov Chain X_n reaches state $g \in E$. Further define $t_g = \infty$ if $X_n \neq g$ for all $n \geq 0$. Then g is accessible from $f \in E$ iff $P(t < \infty | X_0 = f) > 0$.

The property of accessibility is transitive, so that if $f \Rightarrow h$ and $h \Rightarrow g$ then $f \Rightarrow g$. Further, if $f \Rightarrow g$ and $g \Rightarrow f$, then the states communicate ($f \Leftrightarrow g$). If the only equivalence class in the Markov Chain is $f \Leftrightarrow g$ for all $f, g \in E$ then we say that the chain is irreducible.

For all states f in our Markov Chain such that $I_t^1 < s^1$ or $I_t^2 < s^2$, it is required under the rules to reorder to $I_t^1 = s^1$ and $I_t^2 = s^2$. In addition, since $0 < p^1 < 1$ and $0 < p^2 < 1$, it is possible to order inventory such that we re-attain the state $I_t^1 = s^1$

and $i_t^2 = s^1$ from any state where $I_t^1 < s^1$ or $I_t^2 < s^2$. In addition, since $p^1 < 1$ and $p^2 < 1$, it is possible that no failures of Part 1 or Part 2 occur and hence we would reach a state where $I_t^1 = s^1$, $I_t^2 = s^2$, and $v_t^1 = w_t^1 = v_t^2 = w_t^2 = v_t^n = w_t^n = 0$. Moreover, since it is possible that $k=0$, and since $p^{21} > 0$, it is possible that if $I_t^{21} > 0$ that $m_{t+1}^{21} = I_t^{21}$. That is, it is possible that for some period of time \geq lead time that $m_{t+1}^1 = m_{t+1}^2 = 0$ and $m_{t+1}^{21} = I_t^{21}$. In this event we can always return to the state $\{s1, s2, 0, 0, 0 \dots 0, 0\}$ from any state inferior to that state.

Furthermore, for any state where $I_t^1 > s^1$ it is possible to have $m_{t+1}^1 = (i_t^1 - s^1)$ while $m_{t+1}^2 = 0$ and $m_{t+1}^{21} = I_t^1$. As such it is possible to return to $\{s1, s2, 0, 0, 0 \dots 0, 0\}$ from any state superior to that state.

Given the foregoing, it can be seen that $\{s1, s2, 0, 0, 0 \dots 0, 0\}$ is accessible from every state. Since the Markov model states are determined by finding all of the unique destinations reachable by applying the failure rules to $\{s1, s2, 0, 0, 0 \dots 0, 0\}$, it is obvious that all states are accessible from $\{s1, s2, 0, 0, 0 \dots 0, 0\}$. Because of the foregoing, it is clear that all states are communicative and hence the Markov Chain is irreducible. Moreover, it is obvious that there are no absorbing states.

Proof of Aperiodicity

The following rules or variants thereof, are reported heavily in the literature; see for example (Tjims, 2003).

Define: State i of a Markov Chain has a period = $d_p \geq 1$ iff d_p = greatest integer such that $P(n)_{pp} = 0$ if n is not a multiple of d_p .

Given:

1. If $P(n)_{pp} = 0$ for all n then d_p is defined to be ∞ .
2. d_p is defined to be 1 if d_p is not 0 and is not >1 .
3. State p is defined to be aperiodic if $d_p=1$.
4. A Markov Chain is said to be aperiodic if all states are aperiodic.
5. For an irreducible Markov Chain, if one state is aperiodic all states are aperiodic.

From (5), it is sufficient to demonstrate that one state is aperiodic in order to demonstrate that the entire Markov Chain is aperiodic.

We define the fully saturated state to be that state in which all inventory is at the set point and for which there are no outstanding orders and there are no substitutes running and label this as state p . We further define the state q in which all inventory for Part 2 is saturated and no outstanding orders for Part 2 exist and no substitutes are on line but $I_t^1 = s^1 - 1$ and $v_t^1 = 1$. Then there is some non-zero probability of moving from state p to state $q = \Pr(\text{Failures of Part 1} = 1) \Pr(\text{Failures of Part 2} = 0)$. In addition, there is a non-zero probability of moving from state q to state $p = \Pr(\text{Arrivals of Part 1} - \text{Failures of Part 1} = 1) \Pr(\text{Failures of Part 2} = 0)$. This fact shows that $\Pr(d(n)_{pp} \neq 0$ and hence $d_p \neq \infty$.

Next, define state r as any state that is the immediate state occupied prior to a return to state p . For any period greater than 1 there is a positive value probability that $(I_t^p - I_t^r) \neq (\text{Arrivals of Part 1} - \text{Failures of Part 1})$. Given this, it cannot be stated with certainty that any n which required a transit from r to p would occur in the next cycle. As such no value of $n > 1$ has a probability of

occurrence of 1 on any cycle, or any multiple cycle period, let alone on all cycles. Therefore, there is no $n > 1$.

In this case, for state d_p we have $n \neq 0$, $n \neq \infty$, $n \leq 1 \forall \mathbb{Z}$. Hence the state d_p is aperiodic and in conjunction with the fact that the Markov Chain is irreducible we know that all states are therefore aperiodic.

Since the Markov Chain is aperiodic, irreducible, and accessible we can state that it is strongly ergodic and as such the transition probability matrix will in all cases ultimately yield unique steady state values regardless of the initial state vector values.

The Markov Chain Unidirectional Model

Defining the State Sets

We develop the state set universe by beginning with a seed state set that is known to exist in all cases. This seed state set is comprised of that state in which both inventories are filled to the inventory target level and all machines are operational with their primary part. There are no outstanding orders and the substitution level is, of course, zero. This state set is represented as $\{s^1, s^2, 0, 0, 0\}$ in the single day lead scenario and as $\{s^1, s^2, 0, 0, 0, 0, 0\}$ in the two-day lead scenario.

From the seed state, subsequent states are generated by determining each possible state set that can result from using the transformation rules shown in equations 4.1 through 4.6. We first define functions in equations 4.1 through

4.3 that describe the maximum number of failures for each part class and also define the maximum level of substitution.

$$m_j^{1 \max} = n^1 - i_j^{21} + \text{Min}[0, i_j^1] \quad (4.1)$$

$$m_j^{2 \max} = n^2 + \text{Min}[0, i_j^2] \quad (4.2)$$

$$m_j^{21 \max} = i_j^{21} \quad (4.3)$$

$$n_j^{21 \max} = \begin{cases} 0 & \text{if } i_j^1 + v_j^1 - m^1 - m^{21} \geq 0 \\ 0 & \text{if } i_j^2 + w_j^1 - m^2 - m^{21} \leq 0 \\ w_j^1 - p^s = 0 \\ \text{Min}[(i_j^2 + w_j^1 - m^2), m^1 + m^{21} - i_j^1 - v_j^1] & \text{if otherwise} \end{cases} \quad (4.4)$$

If the lead-time for parts delivery is one-day, the seed generates the following branches:

$$U_{j+1} = \begin{matrix} (m_j^{1 \max}) & (m_j^{2 \max}) & (m_j^{21 \max}) & (n_j^{21 \max}) \\ \bigcup_{m^1=0} & \bigcup_{m^2=0} & \bigcup_{m^{21}=0} & \bigcup_{n^{21}=0} \end{matrix} \left[\{(i_j^1 - m^1 - m^{21} + n^{21} + v_j^1), (i_j^2 - m^2 - n^{21} + w_j^1), (i_j^{21} - m^{21} + n^{21}), (s^1 - i_j^1 + m^1 + m^{21} - n^{21} - v_j^1), (s^2 - i_j^2 + m^2 + n^{21})\} \right] \quad (4.5)$$

If the lead-time for parts delivery is two days, the seed generates the following branches:

$$\begin{aligned}
& (m_j^1 \max) (m_j^2 \max) (m_j^{21} \max) (n_j^{21} \max) \\
U_{j+1} = & \bigcup_{m^1=0} \bigcup_{m^2=0} \bigcup_{m^{21}=0} \bigcup_{n^{21}=0} [\{(i_j^1 - m^1 - m^{21} + n^{21} \\
& + v_j^1), (i_j^2 - m^2 - n^{21} + w_j^1), (i_j^{21} - m^{21} \\
& + n^{21}), (v_j^2), (w_j^2), (\text{Max}\{s^1 - i_j^1 + m^1 + m^{21} - n^{21} \\
& - v_j^1, 0\}), (s^2 - i_j^2 + m^2 + n^{21} - w_j^1)\}]
\end{aligned} \tag{4.6}$$

For:

$$\begin{aligned}
\forall \left\{ \begin{array}{l} \{i_j^1 | i_j^1 \text{ is integer on the range } [s^1 - n^1, s^1] \} \text{ iff } \theta = 1 \\ \{i_j^1 | i_j^1 \text{ is integer on the range } [-n^1, s^1 + \text{Min}\{s^2, n^1\}] \} \text{ iff } \theta > 1 \end{array} \right. \\
\forall \left\{ \begin{array}{l} \{i_j^2 | i_j^2 \text{ is integer on the range } [s^2 - n^2, s^2] \} \text{ iff } \theta = 1 \\ \{i_j^2 | i_j^2 \text{ is integer on the range } [-n^2, s^2] \} \text{ iff } \theta > 1 \end{array} \right. \\
\forall \left\{ \begin{array}{l} \{i_j^{21} | i_j^{21} \text{ is integer on the range } [n^1 - s^1, n^1] \} \text{ iff } \theta = 1 \\ \{i_j^{21} | i_j^{21} \text{ is integer on the range } [0, n^1] \} \text{ iff } \theta > 1 \end{array} \right.
\end{aligned}$$

We repeat the foregoing transformations upon the state sets yielded by the previous operation, and we admit to the set of state sets \mathbb{U} only those branches that are not already a member of the universe of state sets. We repeat this process until the branches seed no further unique results.

At this point we have the superset \mathbb{U} comprised of J state sets. Each state set is itself a set comprised of variables which describe the state of one component of the system. The information in each state set belonging to \mathbb{U} comprises enough information from which to determine the next state set without regard to any information from the past. Since each state in \mathbb{U} was generated using only a single prior state exposed to random failure, and substitution possibilities that bear no consideration of past occurrences, along with arrivals known in the set, it is clear that the Markov Property is retained.

For a given state variable, transition from the present value to a future value is determined by the binomial probability of the confluence of events required for that transition within the frame work of rules and assumptions for the model. Similarly, transition from one state set to the next is the cumulative probability of the individual probabilities of transition for each component state variable and the probability of substitution.

Once the super set of state variable sets has been created we next create the transition probability matrix. Each row in the matrix represents the origination state, and each column represents the destination state. For the superset of state sets U of length J , the matrix will consist of J rows by J columns with each row and column running from U_1 through U_J . In our algorithm we begin with the row representing U_1 and systematically apply the transformation rules used to develop the state sets in conjunction with the probability of substitution. The probability of any given transition from a given state set to any other state set is as set forth in equation 4.7.

$$\begin{aligned}
 & \left(\sum_{\alpha=0}^{m_j^1 \max} \sum_{\beta=0}^{m_j^{21} \max} \sum_{\gamma=0}^{n_j^{21} \max} P (i_j^1 - i_{j+\lambda}^1 = m_\alpha^1 + m_\beta^{21} - n_\gamma^{21} - v_j^1) \right) \times \\
 & \left(\sum_{\eta=0}^{m_j^2 \max} \sum_{\beta=0}^{m_j^{21} \max} \sum_{\gamma=0}^{n_j^{21} \max} P (i_j^2 - i_{j+\lambda}^2 = m_\eta^2 + m_\beta^{21} + n_\gamma^{21} - w_j^1) \right) \times \\
 & \left(\sum_{\beta=0}^{m_j^{21} \max} \sum_{\gamma=0}^{n_j^{21} \max} P (i_j^{21} - i_{j+\lambda}^{21} = m_\beta^{21} - n_\gamma^{21}) \right) \times \\
 & (p^s)
 \end{aligned} \tag{4.7}$$

After calculating the transition probabilities we next proceed to calculate the steady state transition probability vector by solving the system of equations

$$\pi = \pi A \quad (4.8)$$

$$1 = \sum_{\alpha=1}^J \pi_{\alpha} \quad (4.9)$$

In essence, our task is to solve for the eigenvector π whose eigenvalue is one. In our algorithm we accomplish this by taking the transpose of the transition probability matrix then replacing the J^{th} row with 1's and dotting the resulting matrix by a vector of $\{\pi_1 \dots \pi_J\}$. We label the resultant matrix A^{modified} . We next replace the J^{th} element in the vector with 1 and label the resultant vector π^{modified} . Then it is only necessary to solve the system of equations such that:

$$\sum_{\beta=1}^J A_{\alpha,\beta}^{\text{modified}} = \pi_{\alpha}^{\text{modified}} \quad \forall \{\alpha | 1 \leq \alpha \leq J\} \quad (4.10)$$

Characterizing the Steady State Values

All equations characterizing the steady state values (which are our measures of goodness) are calculated using the following formulas where $j \in [0, J]$ and $J = \text{number of unique state variable sets}$.

The steady state values for inventory 1, inventory 2, and average substitution level, are very straightforward. We simply multiply the value for that variable in each state variable set by the probability of being in that state variable set and sum the results. That is, we use the weighted average of the state

variable sets where the weighting factors are the steady state probabilities as is shown in equations 4.11 through 4.13.

$$I_{ss}^1 = \sum_{j=1}^J \pi_j i_j^1 \quad (4.11)$$

$$I_{ss}^2 = \sum_{j=1}^J \pi_j i_j^2 \quad (4.12)$$

$$\Delta_{ss} = \sum_{j=1}^J \pi_j i_j^{21} \quad (4.13)$$

Calculation of the steady state order rate for each part is also straightforward; however a small explanation is required. Since all part orders for Part 1 will ultimately manifest as v_j^1 , and all orders for Part 2 will ultimately manifest as w_j^1 , the steady state order rate is the weighted value for v_j^1 and w_j^1 with the weighting factor again being the steady state probability of each state set. Equation 4.14 shows this for Part 1 and equation 4.15 does so for Part 2.

$$v_{ss} = \sum_{j=1}^J \pi_j v_j^1 \quad (4.14)$$

$$w_{ss} = \sum_{j=1}^J \pi_j w_j^1 \quad (4.15)$$

We calculate the fill rate using the percentage of demand we meet completely as that demand occurs. Since each day there is a demand for parts, we consider that we fail to meet demand that day if we do not satisfy that demand in its entirety. Fill rate then is defined in this case as that percentage of

days that we meet demand in its entirety. Since a negative inventory indicates an idle machine, the implementation of this definition for Part 1 and Part 2 is the summation of the steady state probability of each state set in which the inventory of the respective part is not less than zero.

$$F_{ss}^1 = \sum_{j=1}^J \pi_j \forall \{j | i_j^1 \geq 0\} \quad (4.16)$$

$$F_{ss}^2 = \sum_{j=1}^J \pi_j \forall \{j | i_j^2 \geq 0\} \quad (4.17)$$

Total fill rate is similar to the fill rate for an individual part in that total fill rate is defined to be that percentage of days in which we meet all demand. However, in the case of total fill rate, this definition is extended to include the percentage of days in which we do not fail to meet demand for any part. As such, we calculate total fill rate as the summation of the steady state probability of each state set in which the inventory of both parts is not less than zero.

$$F_{ss}^{21} = \sum_{j=1}^J \pi_j \forall \{j | i_j^1 \geq 0 \cap i_j^2 \geq 0\} \quad (4.18)$$

Backorder rate is defined as the weighted average shortage for a particular part. Again, the weighting factor is steady state probability. The calculation of backorder rate for each part type is calculated by summing the product of inventory shortage in a state set with negative inventory times the steady state probability of the respective state set.

$$B_{ss}^1 = \sum_{j=1}^J \pi_j |i_j^1| \quad \forall \{j | i_j^1 < 0\} \quad (4.19)$$

$$B_{ss}^2 = \sum_{j=1}^J \pi_j |i_j^2| \quad \forall \{j | i_j^2 < 0\} \quad (4.20)$$

With these measures of goodness in hand we now proceed to evaluate the sensitivity of the measures of goodness to changes in our decision variables, under varying reliability parameters, in order to examine the benefits of an array of substitution vs. stocking policies.

Algorithm and Model Implementation

The algorithm used to calculate the steady state values for the variables of interest was implemented in Wolfram Mathematica 7.0. The reason we chose Mathematica was that this program allowed us to employ high level programming while utilizing sophisticated pre-built functions for matrix manipulation. Also, Mathematica has no artificial limitations on the size of the matrices it can handle; with the only limitations on size coming from the physical parameters of the computer and operating system. This feature is critical to successfully solving the Markov Chain since we must handle matrices having up to 48,301 rows by 48,301 columns (2,332,986,601 cells).

In our models we use 10 machine type 1 and 10 machine type 2 for both the one-day lead and for the two-day lead problems. We consider s^1 and s^2 over the set $\{1,2,3,4,5\}$ for the one-day lead model and s^1 and s^2 over the set $\{1,2,3,4\}$ for the two-day lead model. In each case we vary the reliability of the substitute

on machine type 1 through the range {0, 0.05, 0.1, 0.15, 0.2, and 0.25}. We vary the reliability of Part 2 on machine type 2 through the same range. In addition to the foregoing, we vary the probability of substitution through the range {0, 0.25, 0.5, 0.5, 0.75, and 1.0}.

We begin the code by defining the domain of critical variables. The variables for probability of substitution and inventory set point are ranged variables that are systematically altered by the algorithm in order to determine the results under a broad array of policies. These include: s^1 , s^2 , and probability of substitution.

In the primary algorithm we use a seed set that represents the state where both primary part inventories are at the target stock level. This state has a value of zero for number of substitutes, and zeros at all values for outstanding orders. With this seed state ({2, 2, 0, 0, 0} for one-day lead and {2, 2, 0, 0, 0, 0, 0} for two-day lead) we then proceed to generate possible new states as branches, as set forth in equations 4.5 and 4.6, using loops over the range of allowable values for each state variable set member. We test each result to ensure it is unique; and if the state set is unique, admit it to \mathbb{U} . The actual coding of this task follows (basestates is the code tag for \mathbb{U}):

```

For[flag = 1, flag ≤ Length[basestates], flag++,
  For[m1 = 0, m1 ≤ n1 - basestates[[flag, 3]] + Min[0, basestates[[flag, 1]]], m1++,
    For[m2 = 0, m2 ≤ n2 + Min[0, basestates[[flag, 2]]], m2++,
      For[m21 = 0, m21 ≤ basestates[[flag, 3]], m21++,
        For[n21 = innerflag[basestates[[flag, 1]], basestates[[flag, 2]], basestates[[flag, 4]], basestates[[flag, 6]], m1, m2, m21],
          n21 ≤ maxn21[basestates[[flag, 1]], basestates[[flag, 2]], basestates[[flag, 4]], basestates[[flag, 6]], m1, m2, m21], n21++,
            i = basestates[[flag, 1]] + basestates[[flag, 4]] - m1 - m21 + n21;
            j = basestates[[flag, 2]] + basestates[[flag, 6]] - m2 - n21;
            k = basestates[[flag, 3]] - m21 + n21;
            v1 = basestates[[flag, 5]];
            w1 = basestates[[flag, 7]];
            w2 = s2 - j - w1;
            v2 = Max[0, s1 - i - v1];
            target = {i, j, k, v1, v2, w1, w2};
            duplicate = 0;
            If[MemberQ[basestates, Part[target, 1]] == False, basestates = AppendTo[basestates, Part[target, 1]]]
          ]
        ]
      ]
    ]
  ]
]

```

Figure 4.1 Filling the State Sets

Once the initial array “basestates” is populated (\mathcal{U}) we proceed to construct a transition probability matrix as a sparse array where each row represents an element of \mathcal{U} , and each column also represents an element of \mathcal{U} . Initially each value in this array is set to zero.

Next, we examine each entry in \mathcal{U} and again apply either equation 4.5 or equation 4.6 using a number of nested loops. This systematically calculates every possible state to which the state set of interest could transition. We then calculate the binomial probability of each transition using equation 4.7. We add the probability for each instance to the appropriate destination column at the row of the member of \mathcal{U} that we are evaluating. The section of code that handles this important task is shown below (A is the transition probability matrix).

stock levels ranging one through five for each part. We examine each combination of the following for the one lead day case and for the two-day lead case.

$$\{s^1 | s^1 \in \{0,1,2,3,4,5\}\}$$

$$\{s^2 | s^2 \in \{0,1,2,3,4,5\}\}$$

$$\{p_t^{21} | p_t^{21} \in \{0, .25, .5, .75, 1\}\}$$

$$\{p_t^2 | p_t^2 \in \{0, .25, .5, .75, 1\}\}$$

$$\{p^s | p^s \in \{0, .25, .5, .75, 1\}\}$$

Analysis of Data from Unidirectional Model

Since we have three decision variables, and a great many measures of goodness (including both inventory levels, order rates for both parts, fill rates for both parts, and backorder rates for both parts), there are a very large number of scenarios we could review from the results of our mathematical modeling. These results are further expanded since we allowed the failure rate of both the Part 2 as a substitute part and Part 2 as a primary part to vary. In order to make our data analysis manageable, we take a systematic approach that examines various potential scenarios with respect to the importance of the measures of goodness, and limit our analysis to those measures of goodness appropriate to that scenario.

First, it is important to note one aspect of our modeling. Since we allow the reliability of the substitute to differ from the reliability of the primary part on machine type 1, and since we substitute with a stochastic approach, conventional

inventory modeling approaches that determine optimum order levels and stock levels break down. In particular, the technique of removing variable costs from the inventory model, while optimizing with respect to holding cost, backorder cost, and fill rate requirements, is not valid because variable costs do not cancel in this model. Moreover, the fact that we are considering service parts makes conventional inventory modeling unreliable for the reasons pointed out in the introduction.

The fact that variable costs cannot be dropped from the optimization modeling has a profound effect upon any policy decision since these costs are very significant when contrasted against holding and backorder costs. In fact, holding and backorder costs are often modeled as a few percentage points of variable cost. As such, small changes in order rate can dominate backorder or holding cost effects. In addition, there is a further complication in this model; since we are dealing with service parts, the inventory costs might be much more significant than normal and the backorder costs could be extreme.

Targeted Order Rate Policies

In the first case we explore a scenario where the primary measure of goodness targeted is the rate of ordering. This scenario is very important in that the cost of parts can significantly outweigh the cost of holding inventory in many scenarios. As such, it makes sense to examine the model's response to decision variable choice as the rate of ordering is held within a target range. As we do this sensitivity analysis, we also examine the effects of varying reliability of Part 2

both with respect to its reliability on machine type two and with respect to its reliability as a substitute on machine type 1.

We began with an examination of the model in which lead times are assumed to be one day. We look at two target order ranges for Part 2: {0.97 - 0.98} and {1.1 - 1.45}.

This scenario reveals that in order to maintain orders in the lowest level range under consideration; we should keep the s^2 at a quantity of one while keeping the s^1 at either two or three. The best total fill rate for any given reliability of p^{21} in this range occurs at a p^s of 0.75. However, it is worth noting that the inventory levels of Part 1 are significantly lower when s^1 is held at two and p^s is 0.5 or 0.25.

Low order rates for Part 2 can be maintained with higher substitution rates under these conditions, but at the cost of higher inventory for Part 1 (inventory increased to 2.01 for Part 1 vs. 1.09 for Part 1 as p^s increased from 0.5 to 0.75). All other measures of goodness are essentially equal, with the exception of the backorder rate of Part 1, which decreases from 0.07 to 0.01 as inventory and substitution rates rises.

When the target ordering level for Part 2 is set to the higher range {1.1 - 1.45} the situation is more complex. In this range we allow increased ordering of Part 2, and increased inventory of Part 2. In exchange, we achieve our ordering goals while maintaining minimum inventory of Part 1. We should note that, as the order rate for Part 2 rises, there is a corresponding decrease in orders for Part 1. That is, we are trading off between orders of Part 1 with orders of Part 2

and optimization will require consideration of the relative cost of the two parts. If Part 1 is significantly cheaper than Part 2, a strategy that minimized Part 2 orders would be sensible. If the inverse is true then the strategy should be reversed.

This case shows clear evidence of the benefits of substitution. If we consider that the trade off of orders for Part 1 vs. Part 2 is factored into policy making based upon the respective cost of the parts, and that as a result the appropriate order target range has been selected, then we would consider all other parameters for differentiation. In all cases of exactly equivalent set points we can observe that when the substitution rate climbs from 0.5 to 0.75 and subsequently to 1, inventory for Part 1 and Part 2 decreases, fill rate for Part 1 and total fill rate increase (while fill rate for Part 2 remains unchanged) and the backorder rate for Part 1 decreases (while backorder rate for Part 2 remains static). In these cases the tradeoff is a decrease in Part 1 purchases as substitution rises and an increase in Part 2 purchases. Thus, in a scenario where Part 2 is similar in price to Part 1 it makes sense to substitute. If Part 2 is more expensive than Part 1 then a closer analysis of the tradeoffs would be required before setting policy.

For all levels of P^{21} the increase in probability of substitution leads to reduced backorder rates for Part 1 and increased fill rates for Part 1. Inventory levels for Part 1 also decrease with increasing probability of substitution. The penalty for increased substitution is always an increased order rate for Part 2. This is counterbalanced by a corresponding decrease in orders for Part 1. The

same notes regarding policy decisions mentioned above for the case apply at all rates of P^{21} .

If we organize the data in such a fashion that we hold the order rate for Part 1 within a tight range instead of holding the order rate for Part 2, we find the results follow the same pattern with respect to sensitivity to P^{21} , s^1, s^2 , and p^s . The main difference in this scenario is that the responsiveness of the dependent variables is less pronounced than in the case of manipulating the independent variables with the order frequency of Part 2 held within a tight range.

As an illustration, consider the case of holding orders of Part 2 to the range {1.1 -1.45} while holding s^1 at 1. We allow p^{21} and s^2 to vary and compare backorder rates for the volatile backorder of Part 1 against these variables as substitution levels change. It is clear that increased levels of substitution decrease backorder of Part 1 at all levels of p^{21} and for all levels of s^2 as can be seen in Figure 4.4.

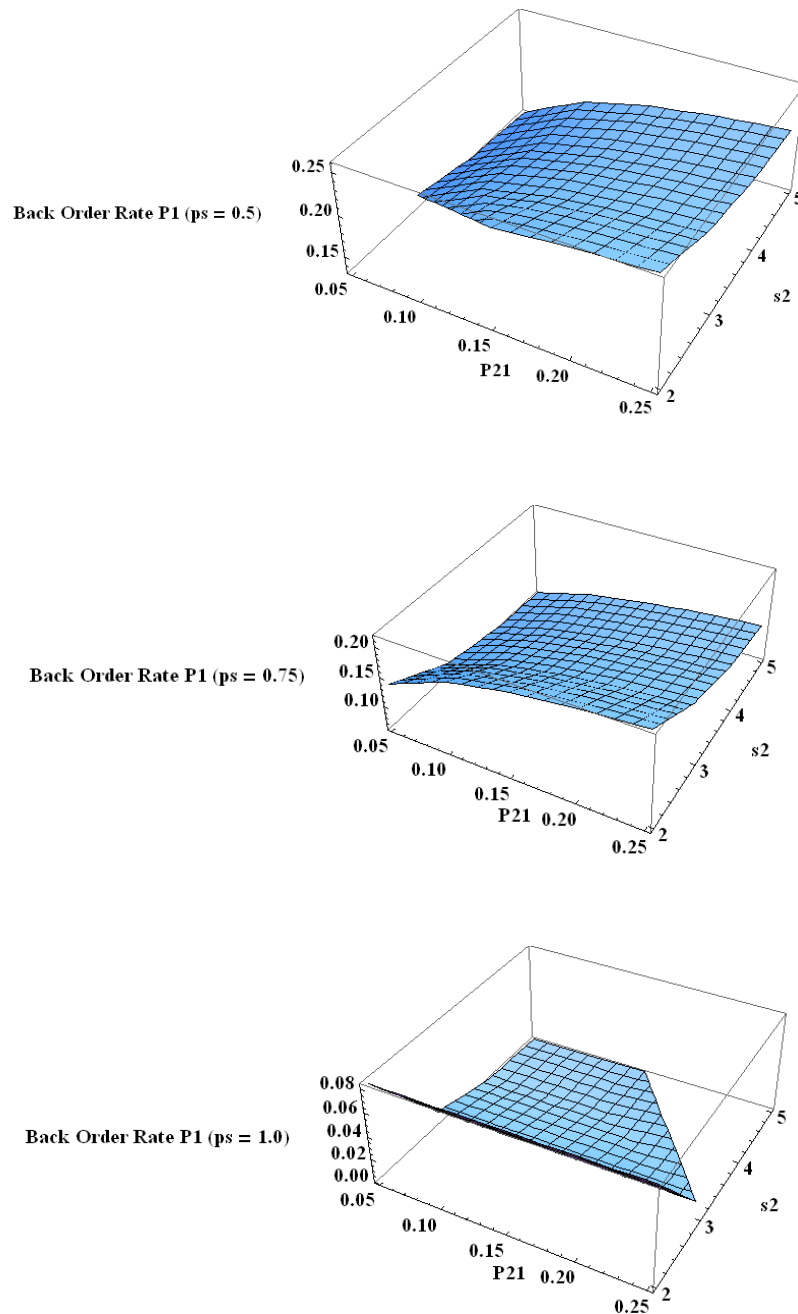


Figure 4.4 Part 1 Backorder Rate as a Function of Part 2 Inventory Set-point and Substitute Reliability For Three Levels of Substitution for Targeted Service Level Policy

When we examine the case where p^{21} varies and we try to achieve targeted ordering levels for Part 1 or Part 2, but increase the lead time for

delivery to two days, we see the same relative relationship between probability of substitution vs. inventory, probability of substitution vs. fill rate, and p^s vs. order level, for the non-controlled part that we saw in the one-day case. However, in the two-day lead model there is an amplified sensitivity of backorder rate as the probability of substitution changes. As p^s rises, the backorder rate for Part 1 at a given set of values for s^1 and s^2 drops rapidly while the backorder rate for Part 2 rises slightly. Given this, there may be more incentive for considering the benefits of substitution as lead time increases if Part 1 Backorder is not significantly less expensive than Part 2 backorder. Again, this must be weighed against the significantly increased order rate for Part 2.

As a demonstration of the magnification in the amplitude of sensitivity just discussed see the Figure 4.2 where we constrain ordering in a two-day scenario for Part 1 in the range {0.5 - .75} against s^1 static at one unit as the reliability of Part 2 on machine 1 varies along with s^2 . Figure 4.5 shows that successively higher probability of substitution leads to significant decrease in the backorder rate for Part 1. Not surprisingly, higher s^2 and lower p^{21} also reduce backorders at all substitution levels.

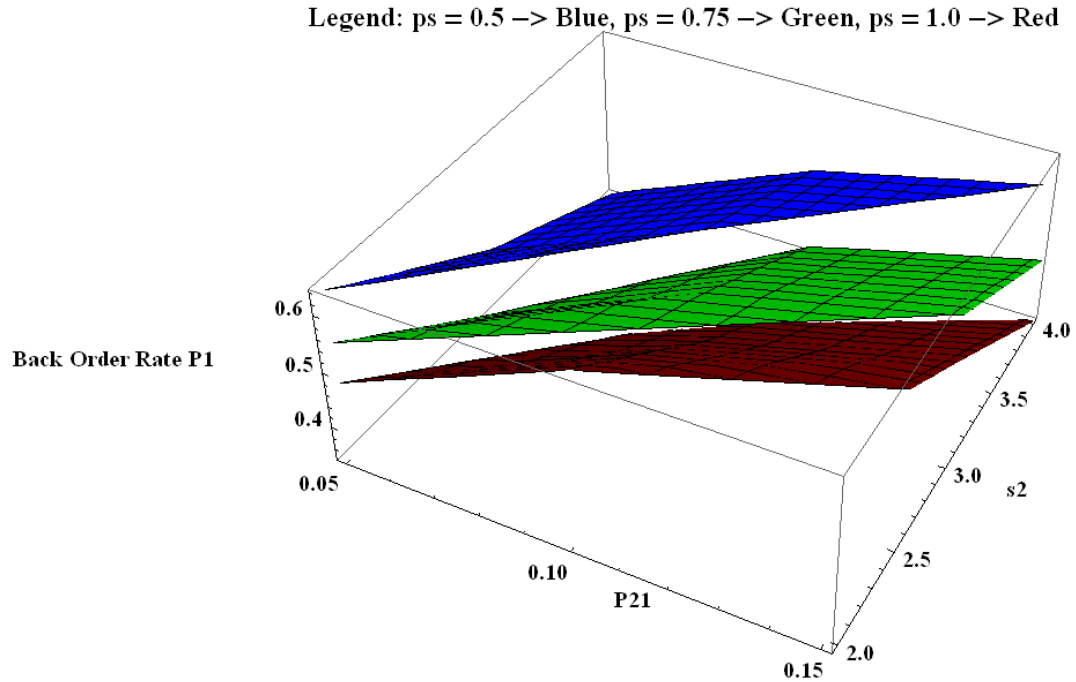


Figure 4.5 Plot of Part 1 Backorder Rate as a Function of Part 2 Inventory Set-point and Reliability of the Substitute for Various Substitution Policies for Targeted Service Level Policy

Again, we examine the two-day lead model with varying s^2 ; however, we now look at the impact of the changing parameters on the backorder rate for Part 2. Worth noting in this case is the fact that the layers are inverted. That is to say, higher substitution rates correlate with higher backorder rates for Part 2. This makes sense because some Part 2 diverted to helping maintain Part 1 operation means that the safety factor at any given level of s^2 is reduced. There are two policy considerations that become apparent when comparing Figure 4.5 (above) with Figure 4.6 (below). First, the decrease in Part 1 backorder with increased substitution is larger than the harm to Part 2 Backorder rates. All things being equal, if uptime across both machines is equally important, or if uptime on machine type 1 is more important than uptime on machine type 2, then

the message is substitution will result in greater customer satisfaction. However, if machine type 2 is more critical, then we must be more careful and perform a cost-benefit analysis.

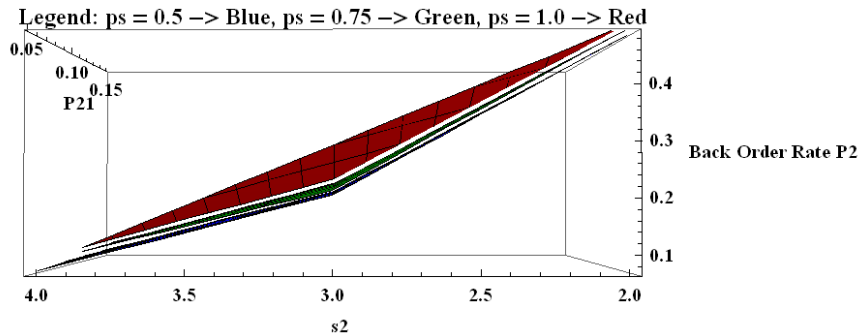


Figure 4.6 Part 2 Backorder Rate as a Function of Inventory 2 Set-point and Substitute Reliability for Targeted Service Level Policy

Targeted Backorder and Fill Rate Policies

The next perspective we explore is where the priority is maintaining control of backorder and fill rate and optimizing upon other measures of goodness while maintaining targeted customer service levels. There are many cases where fill rate and backorder rate would be specified at minimum or targeted levels. One example is case of pollution control systems. Often with these devices, permits require minimum performance with respect to annualized mass of emissions and hours of uptime per year.

Failure to maintain operation of these devices to the levels specified in the permits endangers life and also can result in heavy fines or even plant closure. Typically, when the pollution control for a given process tool is non-operational the equipment discharging to that pollution device must be shut down. Similarly,

in high-value-added operations, or in process bottlenecks, the end user has very little tolerance for downtime and excursions are not acceptable. In these cases the equipment is frequently allotted maximum periods of unscheduled downtime, or down events, and as such backorder and fill rate targets would be common. Despite the foregoing, over specification can lead to extreme costs and as such it is not uncommon to choose an acceptable number for the upper range of the backorder and fill rate operating ranges.

The first targeted backorder rate case we examine is a scenario where backorder of Part 1 is held at {0.1 to 0.2} for the single-day lead scenario. In this example we fix the target stock of Part 1 at one unit and observe the response of the other measures of goodness as we vary P^{21} and s^2 at various levels of substitution. First we examine the effect on inventory of Part 2, as the inventory of Part 1 is relatively inelastic under these conditions. In Figure 4.7 p^s of 0.75 is red and p^s of 0.5 is green. We can see that the increased level of substitution favors lower inventory for Part 2.

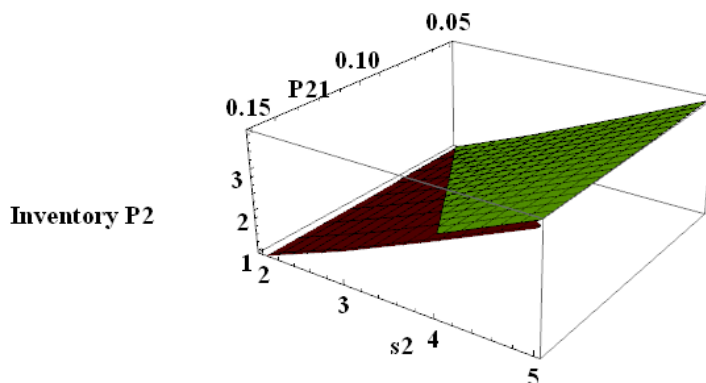


Figure 4.7 Part 2 Inventory as a Function of Part 2 Target Stock and Substitute Reliability on Machine Type 1 for Targeted Service Level Policy

The decrease in inventory of Part 2 with increased substitution is counterbalanced by an increase in the order rate for both Part 1 and Part 2. Figure 4.8 illustrates this increase for orders of Part 1, while Figure 4.9 illustrates this response for Part 2 orders in the one-day lead model when backorder rate of Part 1 is held in the same tight range for the base case scenario ($p^2 = 0.1$, $p^{21} = 0.15$). In this case s^1 is held static at 2 (since its backorder rate is the one being controlled) while p^s and s^2 varied.

It is worth noting that there is an inflection point for both ordering scenarios, which hints at the possibility of optimizing the inventory rate vs. the probability of substitution. Figure 4.8 shows two views to make this inflection point clear for the Part 1 order response, while Figure 4.9 shows the response of the Part 2 order level to the same changes. There are local minima and maxima present for s^2 of 3 and 4 with respect to substitution for orders of Part 1 and local maxima present for the same values with respect to orders for Part 2. If those inventory targets are used then substitution policy should exploit the local minima for cost minimization (minima occur at $p^s = 0.5$) while noting the local maxima for Part 2 ordering and staying away from it (for example at $s^2 = 4$ there is a local maxima for ordering associated with $p^s = 0.75$).

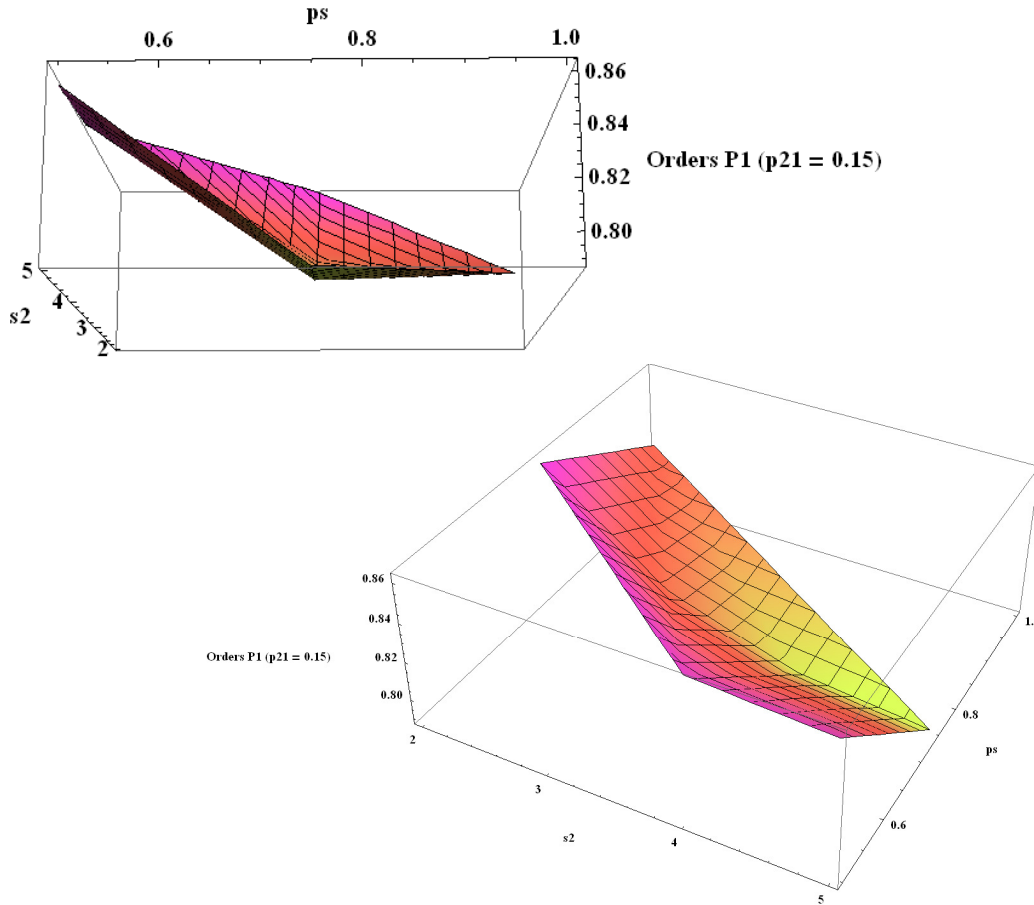


Figure 4.8 Two Views of Part 1 Order rate vs. Substitution Probability and Part 2 Target Stock for Targeted Service Level Policy

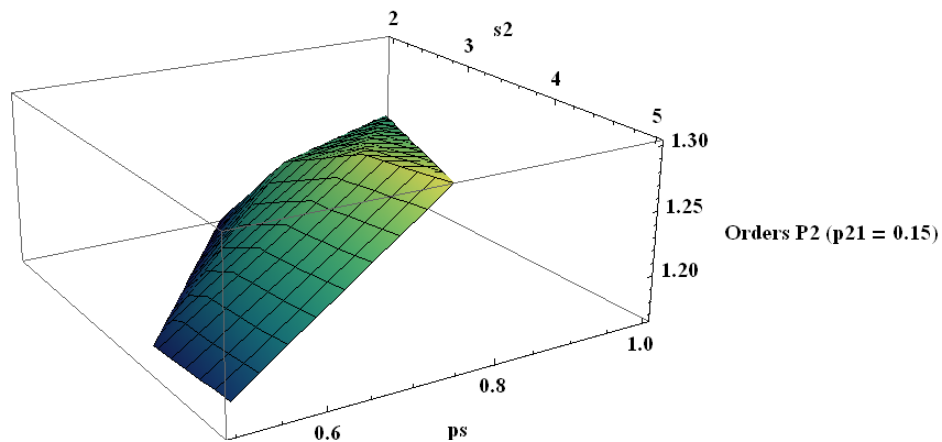


Figure 4.9 Part 2 Order rate vs. Substitution Probability and Part 2 Target Stock for Targeted Service Level Policy

We next explore the single day lead, base case scenario with $p^2=0.1$ and $p^{21} = 0.15$. We examine the sensitivity of inventory to changes in decision variables. We hold s^2 constant at 2 and study the response of inventory to changes in target stock (s^1) for Part 1 and changes in substitution policy.

Figure 4.10 illustrates the effect on steady state inventory as s^1 and the probability of substitution vary for the single day lead case in which backorder of Part 2 is held to the range $\{0.08 - 0.1\}$ with s^2 constant at 2. Inventory of Part 2 remains relatively stable because of the fixed s^2 , and at mid to high levels of s^1 , or if substitution policy dictates a low probability of substitution. At high substitution rates, and for s^1 below 2, there is rapid drop-off in inventory of Part 2 that demonstrates sensitivity to both s^1 and p^s . The reason for this is that the inventory of Part 1 is being held at critically low levels, and with high substitution probability, Part 2 is used to make up for lack of Part 1 inventory that results from the tight inventory strategy for Part 1. Inventory of Part 1 is extremely sensitive to target stock level for Part 1 -- which is expected.

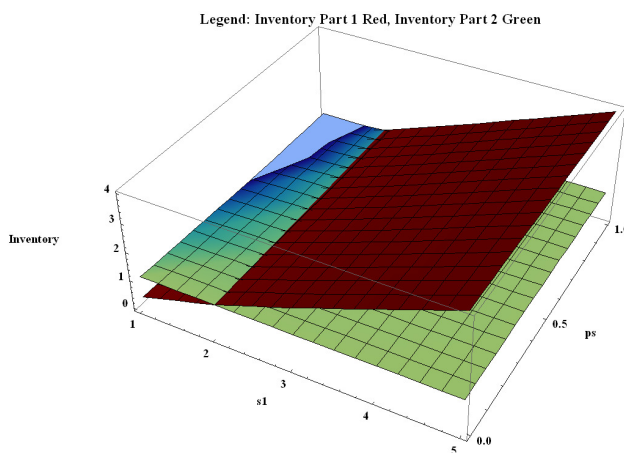


Figure 4.10 Part Inventories as a Function of Substitution Probability and Part 1 Target Stock - One-Day Lead for Targeted Service Level Policy

We modify the previous case to examine the effect that increasing lead time has on the sensitivity of the same parameters. In the two-day lead case backorder rate is higher at all inventory and substitution levels; a byproduct of increased uncertainty that accompanies longer lead times. As such, we choose our target backorder range to include {0.1 - 0.3}. The end result, demonstrated in Figure 4.11, is very similar to the one-day lead model

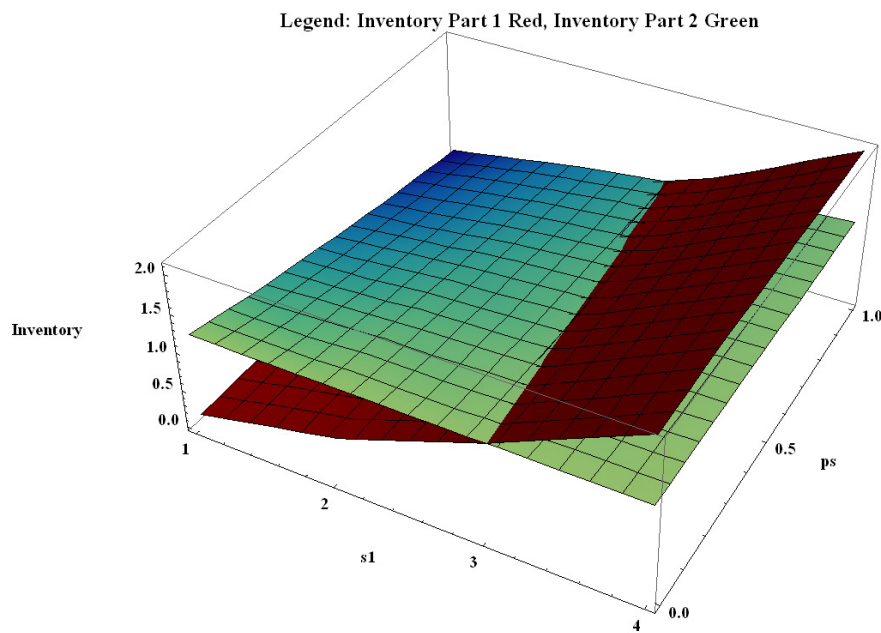


Figure 4.11 Part Inventories as a Function of Substitution Probability and Part 1 Target Stock - Two-day Lead for Targeted Service Level Policy

The experiment explores a direct comparison of the sensitivity of one backorder rate when the other part's backorder rate is constrained to a fixed range. In this case, we examine the single-day lead model with p^2 at 0.1 and consider the impact of changes in reliability of the substitute, changes in substitution rate, and changes in target stock level, for the independent

backorder rate (in this case that of Part 1). Figure 4.12 compares the backorder rate for Part 1 when backorder level for Part 2 is held in the tight range of {0.08 – 0.1} with s^2 constant at 2. Not surprisingly, backorder for Part 1 climbs rapidly with the decrease in s^1 . Also of interest is the relationship of backorder rate for Part 1 to p^{21} ; at high substitution probabilities, and high inventory targets for Part 1, the backorder rate is not sensitive to changes in p^{21} . However, as substitution probability increases, and s^1 drops, the backorder rate of Part 1 becomes slightly sensitive to the reliability of the substituted part.

There is an obvious impact on backorder rate as the probability of substitution increases; higher substitution always correlates with lower Part 1 backorder levels – this confirms the primary motivation of substitution – that is, reducing backorder levels of Part 1.

Legend: $p_s = 0 \rightarrow$ Red, $p_s = 0.25 \rightarrow$ Green, $p_s = 0.5 \rightarrow$ Brown, $p_s = 0.75 \rightarrow$ Orange, $p_s = 1 \rightarrow$ Blue

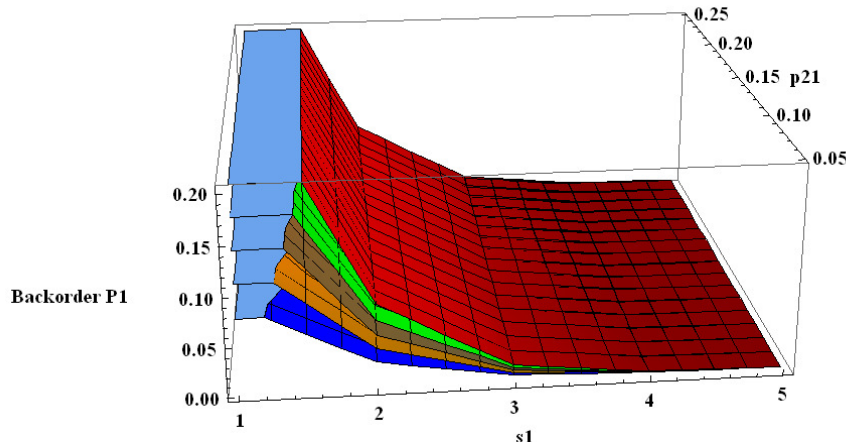


Figure 4.12 Backorder rate of Part 1 vs. Target Stock Part 1 and Substitute Reliability at Under Various Substitution Rates – One-Day Lead for Targeted Service Level Policy

Again, we examine the sensitivity of the inventory of Part 1 to changes in our decision variables; however, we add the dimension of longer lead and see how that alters the response by examining the two-day lead scenario as is illustrated in Figure 4.13. We consider the two-day lead case where s^2 is held at 3 and backorder of Part 2 is held in the $\{.1 - .3\}$ range. We see a similar result in the two-day scenario as in the one-day scenario; but both the benefits of substitution, and the effects of the reliability of the substitute, have a more marked impact than in the case of the one-day model. This further demonstrates the fact that substitution is a more important influence as the uncertainty – in this case in the form of increased lead time – grows.

Legend: $ps = 0 \rightarrow$ Red, $ps = 0.25 \rightarrow$ Green, $ps = 0.5 \rightarrow$ Brown, $ps = 0.75 \rightarrow$ Orange, $ps = 1.0 \rightarrow$ Blue

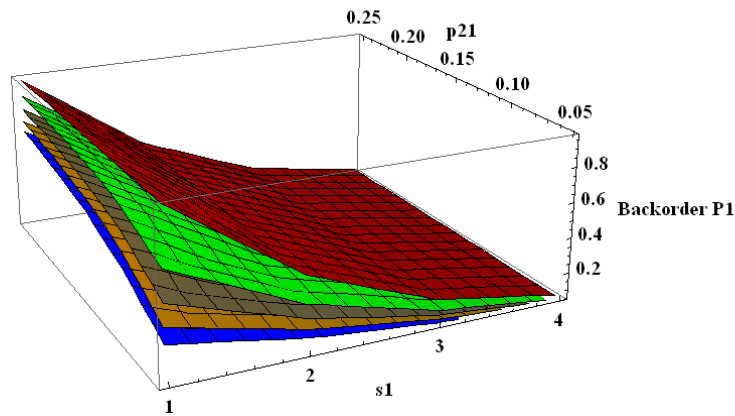


Figure 4.13 Backorder rate of Part 1 vs. Target Stock Part 1 and Substitute Reliability at Under Various Substitution Rates – Two-day Lead for Targeted Service Level Policy

In the next case we examine a slightly different perspective on customer satisfaction and look at a constrained fill rate policy. Backorder and fill rate are similar measures of goodness in that policies focused on either strive to maintain

uptime and customer satisfaction. In addition, in many instances, backorder and fill rate measures of goodness move together with respect to manipulation of the decision variables and changes in parameters. There are important differences however. Backorder can be viewed as a measure of the intensity of badness. That is, high backorder rates correlate with multiple down systems, and extended downtime, while fill rate can be viewed as a measure of the frequency of badness. We can experience high backorder rates if we have a relatively small frequency of occasions in which a large number of machines go down each time there are failures. This could equate with a high fill rate if the frequency of these events is low. Conversely, we could have a poor fill rate with a reasonable backorder rate if there are a large number of events where a single machine is down for a short period of time.

It is relatively easy to understand why a firm would target backorder rate as a control in order to ensure customer satisfaction, revenue maximization, and regulatory compliance. If tools are down then all of the foregoing suffer. However, fill rate may be a dominant concern and key target, if any excursion, albeit for a short time, has unacceptable consequences. The simplest reasoning behind targeting fill rate is that if customers see a frequency of problems they will associate the product with poor quality and perceive a lack of reliability. Other examples of reasons to target fill rate levels include cases where a failure is associated with immediate and irrevocable harm.

In the following example we study a case where the target fill rate of machine type 1 is benchmarked to maintain a certain fill rate level and search for

the policy or policies that optimizes the fill rate for machine type 2 demand. We explore a case where Part 1 fill rate is required to be greater than 80 percent and examine the Part 2 fill rate at various target stock levels for Part 1 and Part 2. This analysis considers a two-day lead for parts delivery using the base case reliabilities for Part 2 where $p^2 = 0.1$ and $p^{21} = 0.15$.

What we observe in this comparison, shown in Figure 4.14, is that increased probability of substitution results in slightly depressed fill rates for Part 2 demand by machine type 2. This demand is most pronounced at low target inventory levels for Part 1 and then becomes a stronger factor as target inventory of Part 2 rises. This again, is a reaffirmation that Part 2 measures of goodness diminish when we use Part 2 stock to improve Part 1 measures of goodness.

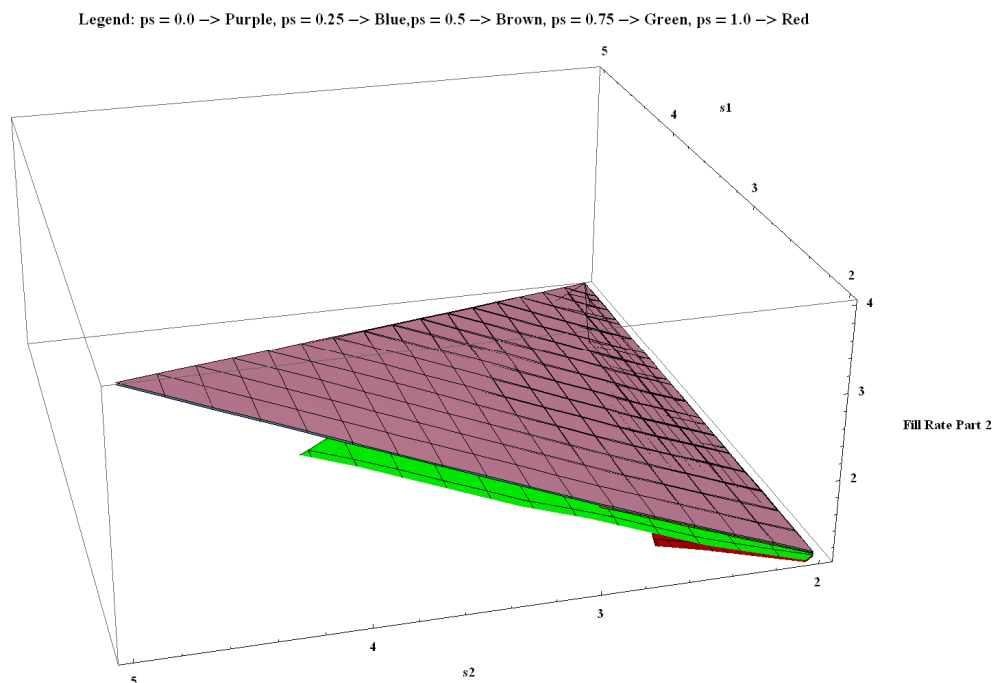


Figure 4.14 Fill rate of Part 2 as a Function of Target Stock Levels at Various Substitution Rates for Targeted Service Level Policy

Targeted Inventory Policies

There are many cases where inventory size is a key constraint as well as a measure of goodness. For example, in cases where the inventory is very expensive, the cost of capital associated with holding even moderate levels of stock can be prohibitive. An example of this type of inventory constraint is the inventory parts for a nuclear reactor. In other cases, the inventory item might be perishable or very near obsolescence. Another factor that might make inventory a constraint is very limited storage area such as is often the case when customers allocate on-site areas for storage of a vendor's service parts.

Finally, the part might itself be dangerous and storage of large quantities of the component may be very risky or even illegal. Many chemicals fall into this classification (in fact, the facilities' hazardous materials business plan will put firm limits on the storage of these chemicals in the United States). In the exploration of what occurs in substitution when inventory levels are constrained we attempt to develop a policy to maximize performance through our key measures of goodness with targeted inventory some interesting results occur.

In this case we fix the target stock for Part 1 very low as might be the case when Part 1 is a part that must be kept low. A good example for this case would be if Part 1 were a dangerous chemical that is restricted under the Toxic Substances Control Act (TSCA) but Part 2 was a non-restricted chemical. We hold inventory for Part 1 low at a steady state value of {0.15 – 0.35} units. In order to achieve this we set a target stock level for Part 1 of one unit. In the two-day lead model we find that we cannot achieve these inventory levels without

substitution so we compare substitution probabilities ranging from {0.25 - 1.00} and observe the response of inventory 1 to changing reliability of the substitute and changing target stock levels for Part 2.

The results of this experiment are presented in figure 4.15. It is clear that the inventory of Part 2 is quite sensitive to substitution probability as successively lower substitution levels result in significantly increased inventory. In addition, lower levels of substitute reliability result in greater consumption of Part 2 as a substitute and this effect is amplified at higher levels of substitution.

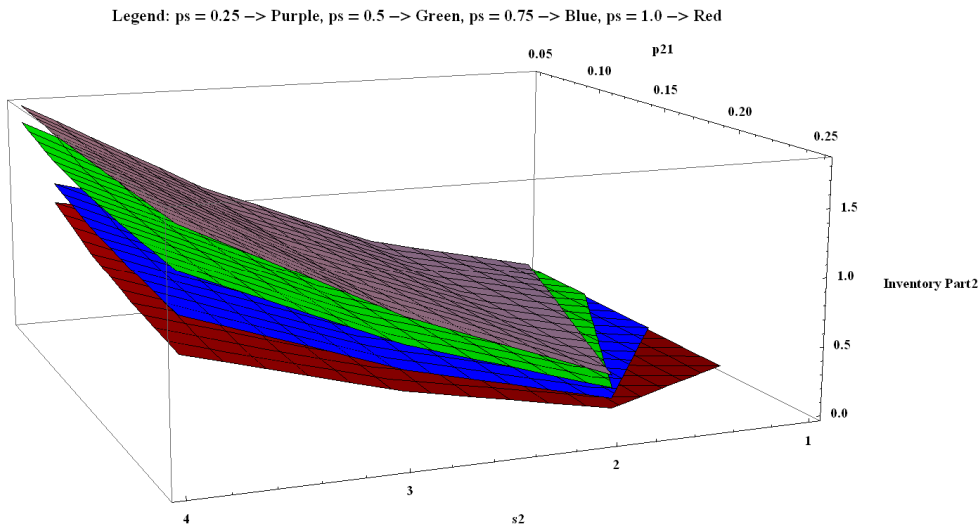


Figure 4.15 Inventory of Part 2 as a Function of Substitute Reliability and Part 2 Target Stock Level – Two-Day Lead for Constrained Inventory Policy

To compare the effects of shorter lead time we will look at the same example as for the one-day lead case; however, in this case the range targeted for inventory 1 has to be slightly higher due to the fact that depletion of inventory at any given target is lower; hence we do not find an acceptably large sample in

the range used for the two-day case at the target stock level of one unit for Part 1. The response of Part 2 inventory levels to changes in decision variables and parameters is similar to the previous example. However, this response is far less sensitive to substitution rate and the reliability of the substitute than was the case in the two-day lead model, as is shown in Figure 4.16. This response makes perfect sense in that the increased uncertainty in ordering seen in the in the two-day model requires heavier substitution so there is greater sensitivity to the substitute’s reliability in the two-day lead model than is the case in the single day lead model

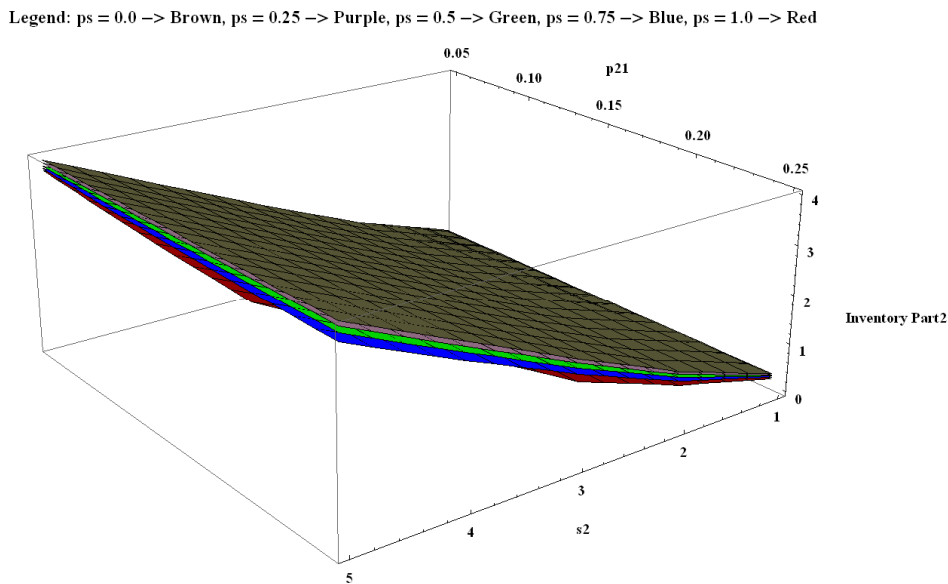


Figure 4.16 Inventory of Part 2 as a Function of Substitute reliability and Part 2 Target Stock Level – One-Day Lead for Constrained Inventory Policy

The next permutation of the static inventory policy case that we investigate is the response of order levels to changes in the target inventory of Part 2, and to

changes in the probability of substitution, when the inventory of Part 1 is constrained to a tight target. First, we look at the two-day lead scenario with s^1 fixed at one unit and steady state inventory of Part 1 kept within the range 0.15 units to 0.30 units. For this examination we review the base case probabilities for reliability of Part 2 on both machine type 2 and machine type 1.

The results of this examination demonstrate that the order rate of Part 1 is very sensitive to both the probability of substitution and to the inventory set point for Part 2. As shown in Figure 4.17, orders for Part 1 climb as s^2 drops, and orders for Part 1 drop as the probability of substitution rises. The increase in Part 1 orders with decreasing substitution results from the fact that Part 1 is being used to satisfy a greater portion of machine type 1 demand. Similarly, the increase in Part 2 inventory target means that more Part 2 is available when a substitution opportunity presents itself and as such there is a larger portion of machine type 1 demand filled by Part 2.

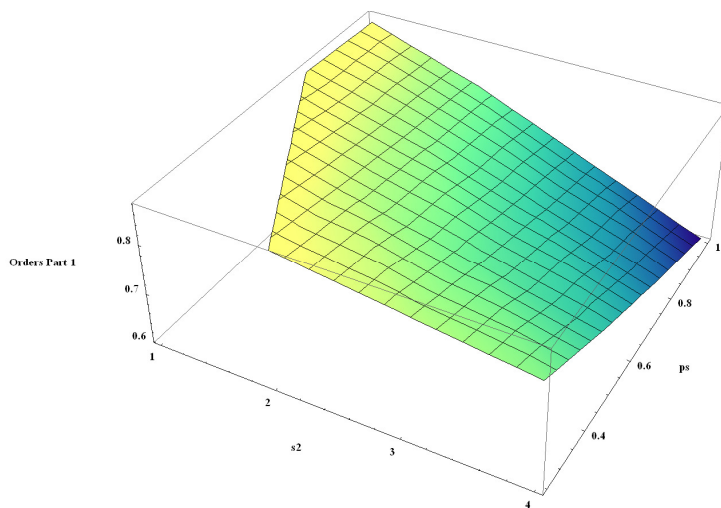


Figure 4.17 Order Rate Part 1 as a Function of Substitution rate and Part 2 Target Stock Constrained Inventory Policy with 2 Day Lead

Using the same model parameters, we next examine the response of the Part 2 order rate. The plot of order rate for Part 2 vs. Part 2 target stock and probability of substitution shown in Figure 4.18 is nearly a mirror image of the plot for Part 1 orders. Orders for Part 2 rise as s^2 rises and as the probability of substitution rises. This is again due to the fact that increases in target stock (and hence inventory) for Part 2, and increases in the probability of substitution, lead to increased usage of Part 2 to fill machine type 1 demand.

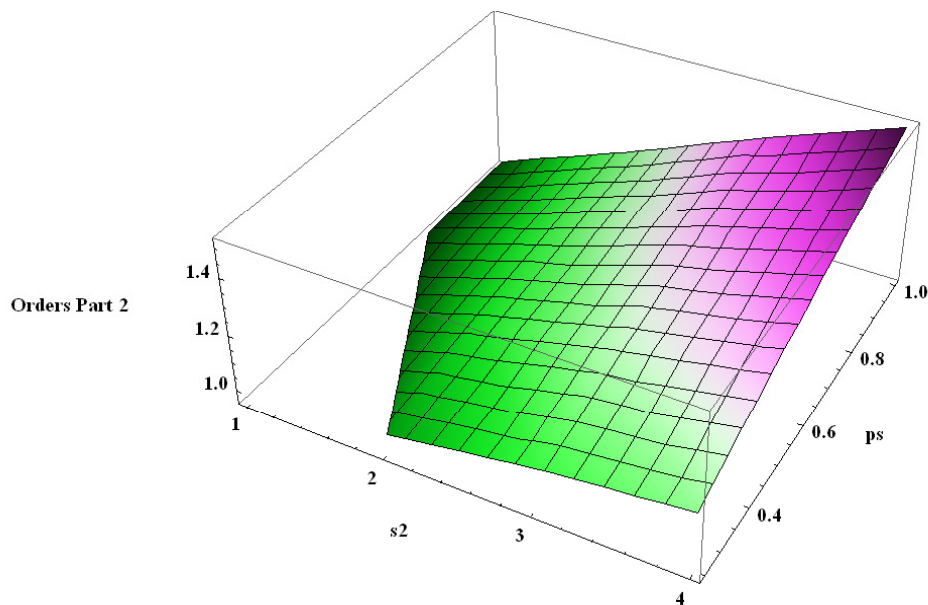


Figure 4.18 Order Rate Part 2 as a Function of Substitution rate and Part 2 Target Stock - Two-day Lead Constrained Inventory Policy

We next look at a scenario that concentrates on customer satisfaction when inventory for Part 1 is targeted. The first case is the one-day lead model with a narrow target range for inventory 1 between 0.25 units and 0.45 units. We constrain the inventory target for Part 1 (s^1) at one and allow s^2 to vary. Figure

4.19 shows the steady state total fill rate vs. s^2 as the reliability of the substitute ranges from 0.05 to 0.25.

We can observe that higher substitution rates correlate with higher total fill rate at all inventory levels, and all reliabilities, for the substitute. This result is simply because when we choose not to substitute, although substitution is possible, then we are choosing to allow a machine type 1 to experience a downtime that it would not experience if we opted to substitute.

Legend: $ps = 0.0 \rightarrow$ Violet, $ps = 0.25 \rightarrow$ Blue, $ps = 0.5 \rightarrow$ Green, $ps = 0.75 \rightarrow$ Brown, $ps = 1.0 \rightarrow$ Red

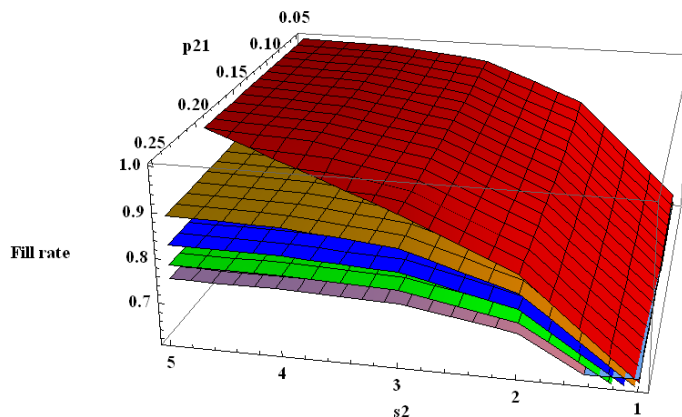


Figure 4.19 Fill Rate as a Function of Substitute Reliability and Part 2 Target Stock at Various Substitution Rates - One-Day Lead Constrained Inventory Policy

This chart also makes it clear that the reduction to Part 2 inventory from reallocation of stock to fill machine type 1 demand does not reduce Part 2 fill rate to the extent that overall fill rate is reduced.

Relative Part Reliability

One factor that bears examination when considering substitution policy is the relative reliability of the parts involved in the exchange. That is, we are curious as to what are the ramifications on substitution policy when the part that can serve as a substitute is more or less reliable in its native application than the part for which it substitutes. In order to explore this area we examined performance with a wide array of reliability ratios for Part 2 with respect to Part 1. Specifically, we considered $p^2 = \{0.05, 0.1, 0.15, 0.20, 0.25\}$.

First we examine the customer service based measure of goodness, total fill rate, at various Part 2 target stock levels and various values for Part 2 reliability as a primary part on machine type 2. In this examination we hold the target stock level for Part 1 at one because we really want to observe the impact of substitution and Part 2 reliability upon customer service levels. Were we to allow s^1 to be too high, it would mute the responses of independent variables to changing decision variables and parameters. We looked at the two-day parts lead model and the results of our experiment are shown in Figure 4.20.

What we observe in this scenario is that substitution, as in earlier experiments, benefits total fill rate because failure to substitute when an opportunity to do so presents itself, must impact fill rate. We also see that total fill rate rises with increased s^2 and rises with increased reliability of Part 2.

The unique information from this experiment is that the performance increase on fill rate levels for high substitution probability policies increases with increased relative reliability of Part 2 and also increases with increased s^2 .

Examination of Figure 4.21, an alternate view of the same experiment, reveals that this advantage is not linear, but rather grows in intensity as p^2 improves and as s^2 grows. This response occurs due to the fact that higher levels of s^2 provides a cushion to compensate for the reallocation of parts to machine type 1 which would otherwise reduce the ability to meet demand on machine type 2. Higher reliability of Part 2 on machine type 2 means that the demand from machine type 2 is smaller and increases the likelihood that any given level of s^2 will meet demand.

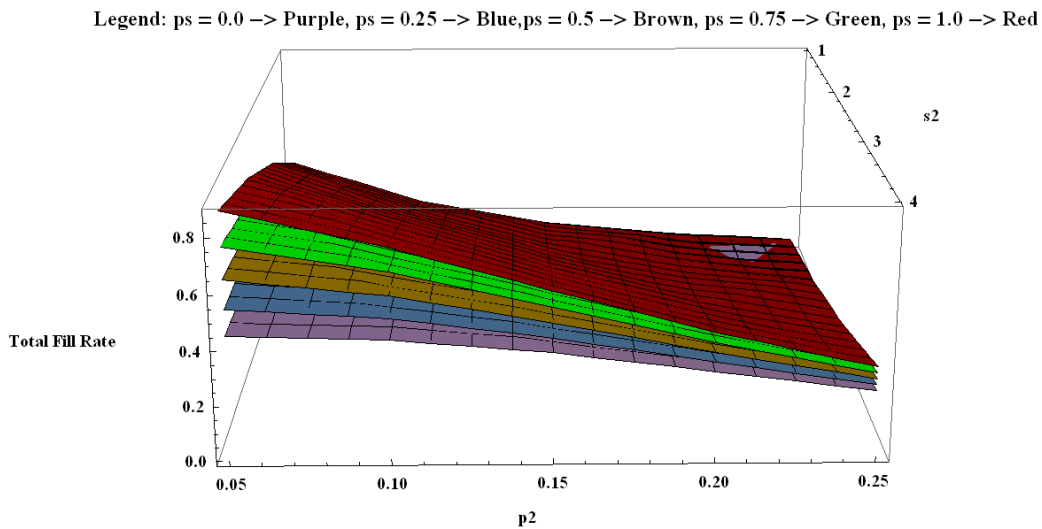


Figure 4.20 Total Fill Rate as a Function of Part 2 reliability as a Primary Part and as a Function of Part 2 Target Stock for Various Substitution Rates - Two-day Model with Part 1 Target Stock Constrained

Legend: $ps = 0.0 \rightarrow$ Purple, $ps = 0.25 \rightarrow$ Blue, $ps = 0.5 \rightarrow$ Brown, $ps = 0.75 \rightarrow$ Green, $ps = 1.0 \rightarrow$ Red

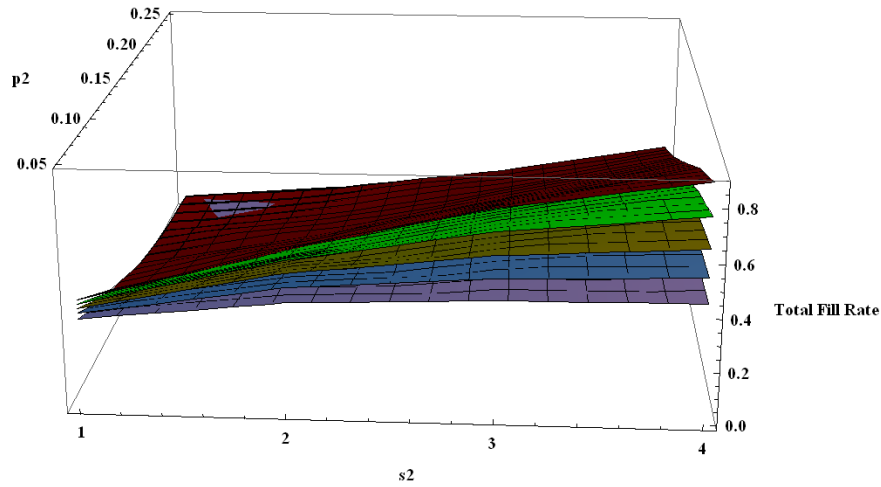


Figure 4.21 Alternate View of the Plot in 4.20

Next, we target the order level for Part 2 in the single day lead case on the range $\{1.4 - 2.0\}$, while keeping s^1 static at two units of inventory, and examine the response of the backorder rate for Part 1 as substitution policy and reliability of Part 2 varies. What we observe in Figure 4.22 is that the backorder rate for Part 2 varies. What we observe in Figure 4.22 is that the backorder rate for Part 2 is, not surprisingly, very sensitive to s^2 . In addition, it is very apparent that backorder rate of Part 2 is very sensitive to the reliability of Part 2 on machine type 2. Very small changes in reliability result in large changes to the backorder rate. As the failure rate of Part 2 on machine type 2 moves from 0.15 to 0.20, the backorder rate of Part 2 climbs 67 percent. Finally, as observed in earlier cases, substitution of Part 2 onto machine type 1 increases the backorder rate for Part 2 because Part 2 inventory is reallocated to meet machine type 1 demand and thus less available to meet machine type 2 demand.

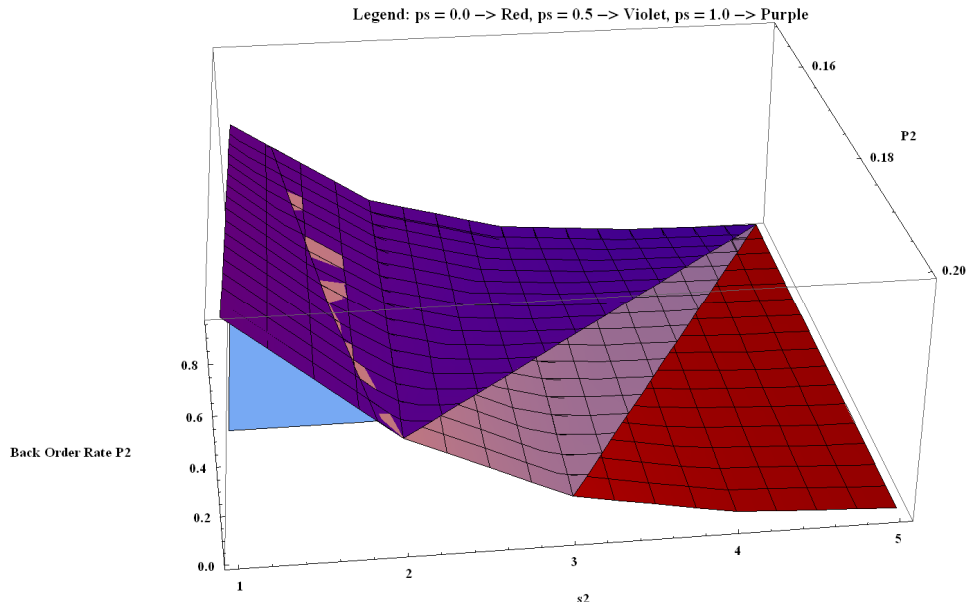


Figure 4.22 Backorder Rate for Part 2 as a Function of Part 2 Reliability as a Primary Part and as a Function of Part 2 Target Stock Levels for Targeted Part 1 Ordering at Various Substitution Rates - One-Day Lead

We conclude our evaluation of Part 2 reliability as a driver of policy with an examination of the impact of p^2 changes on ordering. Figure 4.23 demonstrates the two-day lead scenario where s^1 is constrained to one unit and s^2 is fixed at two units. In this case, we explore the sensitivity of ordering for each part type vs. changes to Part 2 target inventory and probability of substitution. What we observe in this trial is that Part 2 ordering is very sensitive to Part 2 reliability and slightly sensitive to Part 2 target stock level. This can be explained by fact that higher failure rates of Part 2 on machine type 2 obviously require more part orders in order to restock the more frequent failures. Setting higher stock targets for Part 2 means it is more likely that Part 2 will be available for substitution and hence more likely to be backfill for Part 1 which effectively increases the demand

rate for Part 2. Part 1 ordering is slightly sensitive to Part 2 reliability and slightly sensitive to Part 2 stock levels. The reasons for the Part 1 ordering sensitivity include the fact that greater failures of Part 2 in its native application make it less likely that Part 2 will be available as a potential substitute and hence more demand for machine type 1 will fall upon Part 1 inventory. Conversely, increased stock targets for Part 2 make it more likely that Part 2 will be available to meet machine type 1 demand as a substitute and this ultimately reduces the demand for machine type 1 filled by Part 1 orders.

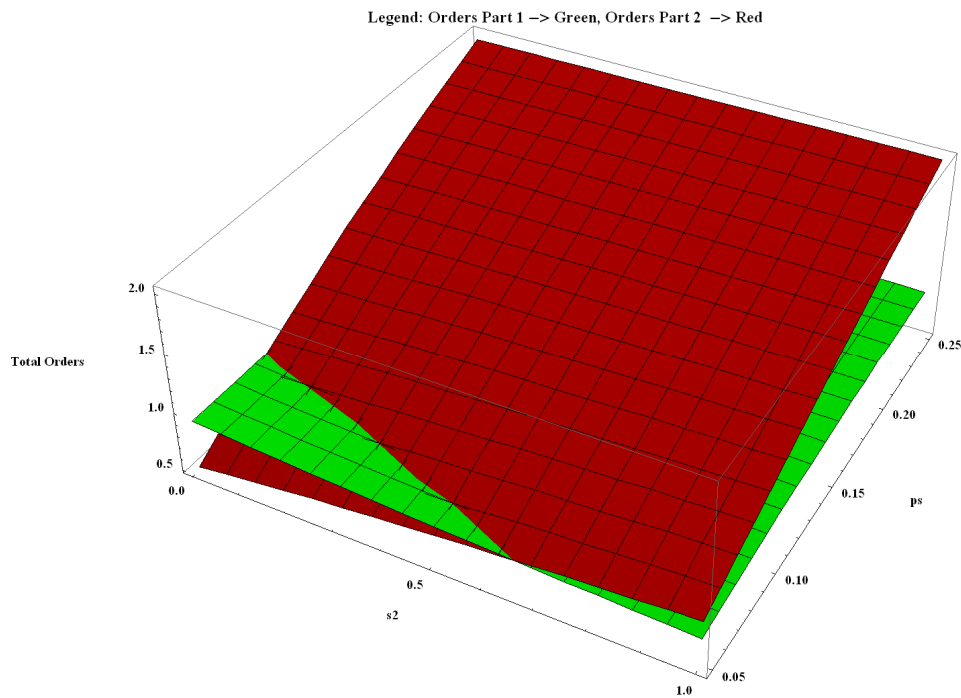


Figure 4.23 Total Order Rate for Each Part Type as a Function of Part 2 Reliability as a Primary Part and as a Function of Substitution Rate for Targeted Part 1 - Two-day Lead

Approximation Method for Larger Models

The algorithm we use to build a transition probability matrix, and solve for the steady state vector, demonstrates that a Markov Chain is a very powerful tool for developing stocking and substitution policies. In addition, this work has demonstrated that the myopic model in which single-period results are used to develop policy has serious limitations. However, as we have seen in this model, as the lead time grows larger the size of the state set needed to contain the information on the state of the system grows rapidly. As a result, the size of the matrices needed to handle the problem quickly demand more computing resources and computing time than is reasonably available. The demands on computing resources also grow rapidly as the population of machines grows. Cohen and Barnhart (2006) demonstrated that these problems can become unsolvable when the number of machine classes or number of parts grows. Therefore it is desirable to look for approximation methods in order to successfully approach many real world applications of this research.

The usefulness of modeling increased lead time for larger numbers of machines, coupled with the computational difficulty of doing so, led us to explore methods of reducing calculation intensity. We developed a method that significantly reduced the computational complexity and we call this method the Boycott Method.

The Boycott Method exploits the fact that a large number of states may have some non-zero probability of occurring but are so statistically unlikely that they could be neglected without noticeably altering the steady state values, or

our policy decisions, as long as ignoring these highly unlikely states did not interfere with our ability to find the steady state vector.

In the Boycott Method we enumerate the state sets through an entire iteration of the algorithm, and those state sets that have a probability less than the boycott threshold are added to a boycott list to be excluded in future passes through the algorithm with higher values of target inventory. This excludes those state sets below the boycott threshold and child states derived from these boycotted states. The end result is a significantly reduced universe of state sets as target inventory grows and this helps to slow the growth of the transition probability matrix with higher target inventory. This also reduces the number of passes through the nested loops needed to process each state variable set in the state variable set universe.

The obvious question when implementing a method such as the Boycott Method is how small can a number be before we can safely ignore it? A starting point is the machine epsilon for the computing system performing the calculation. Machine Epsilon is that number that when added to 0 is indistinguishable from zero because of binary number storage. On most personal computers that number is of the order of magnitude of 10^{-16} . Clearly numbers smaller than machine epsilon are of little utility in performing any calculation. Carrying such numbers greatly increases the size of memory required to calculate large Markov Chains and severely interferes with exploiting the advantages of sparse array processing.

As a practical matter, the boycott threshold can be considerably larger than the machine epsilon without impacting calculation. In our experiments we found that a boycott threshold of 10^{-8} yielded excellent results as is shown in the following test case.

We conducted an experiment in which we considered a system with a one-day lead and the following values for parameters and decision variables

$$p_1^1 = 0.1$$

$$p_1^2 = 0.15$$

$$p_2^2 = 0.1$$

$$p^s = \{0, 0.25, 0.5, 0.75, 1\}$$

$$s^1 = \{1, 2, 3\}$$

$$s^2 = \{1, 2, 3\}$$

$$n^1 = 5$$

$$n^2 = 5$$

Paired t-test Validation for Approximation Method

We ran this model for a scenario in which no boycotting occurred and for a model in which a boycott list was used. Upon completion of this model we generated all of the measures of goodness as normal and we calculated the difference between each pair of data generated by the non-approximation method and the approximation method. Before proceeding with a paired t-test we subjected the differences in each paired set of variables to an Anderson-Darling test for normality to ensure that the differences in the pairs were normally

distributed. For each measure of goodness we found that the p-value for the normality test was <0.005 and hence we could not reject the null hypothesis that the differences in the pairs were normally distributed at the 99% confidence level.

Since we were comfortable that our data would meet the conditions needed to use a paired t-test we proceeded with that test for each measure of goodness. We paired the data for each measure generated by the approximation method, with the corresponding data generated by the non-approximation method. We next used a paired t-test to determine if there was a statistically significant difference in the data produced by each model. Table 4.2 shows the results of the statistical testing. It is clear that the approximation method produces results that are very close to the results from the standard (non-approximation) method (in the test results, the presence of “NS” in front of the variable indicates it is from the non-approximated model).

In evaluating the results, we first note that for all of comparisons the standard deviation (and hence the variance) is very nearly identical, and often identical, in each test. We further note that the 95% confidence intervals include zero. In fact, zero is very near the mean for each case. The results in Table 4.2 clearly indicate that we cannot reject the null hypothesis, which is that the mean of the difference in steady state values between the non-approximation method and the standard method is zero at the 95% confidence level.

Table 4.2 Results of Paired t-test Comparing Approximation Method to Non Approximation Method

Steady State Measure	Paired t-test Result ($H_0: \mu_{\text{difference}} = 0$)				
Inventory Part 1	N	Mean	StDev	SE Mean	
	NSI1	45	1.16445	0.67670	0.10088
	I1	45	1.16440	0.67672	0.10088
	Difference	45	0.000049	0.854640	0.127402
95% CI for mean difference: (-0.256713, 0.256812) T-Test of mean difference = 0 (vs not = 0): T-Value = 0.00 P-Value = 1.000					
Inventory Part 2	N	Mean	StDev	SE Mean	
	NSI2	45	1.09354	0.66050	0.09846
	I2	45	1.09355	0.66052	0.09846
	Difference	45	-0.000010	0.851100	0.126874
95% CI for mean difference: (-0.255709, 0.255689) T-Test of mean difference = 0 (vs not = 0): T-Value = -0.00 P-Value = 1.000					
Fill Rate Part 1	N	Mean	StDev	SE Mean	
	NSF1	45	0.924437	0.079636	0.011871
	F1	45	0.924443	0.079641	0.011872
	Difference	45	-0.000007	0.083035	0.012378
95% CI for mean difference: (-0.024953, 0.024940) T-Test of mean difference = 0 (vs not = 0): T-Value = -0.00 P-Value = 1.000					
Fill Rate Part 2	N	Mean	StDev	SE Mean	
	NSF2	45	0.889385	0.099873	0.014888
	F2	45	0.889396	0.099875	0.014888
	Difference	45	-0.000010	0.129940	0.019370
95% CI for mean difference: (-0.039049, 0.039028) T-Test of mean difference = 0 (vs not = 0): T-Value = -0.00 P-Value = 1.000					
Total Fill Rate	N	Mean	StDev	SE Mean	
	NSF12	45	0.826190	0.122856	0.018314
	F12	45	0.826201	0.122866	0.018316
	Difference	45	-0.000012	0.144771	0.021581
95% CI for mean difference: (-0.043506, 0.043483) T-Test of mean difference = 0 (vs not = 0): T-Value = -0.00 P-Value = 1.000					

Table 4.2 continued

Steady State Measure	Paired t-test Result ($H_0: \mu_{\text{difference}} = 0$)				
Backorder Rate Part 1		N	Mean	StDev	SE Mean
	NSB1	45	0.092016	0.099969	0.014903
	B1	45	0.091987	0.099970	0.014903
	Difference	45	0.000028	0.103988	0.015502
95% CI for mean difference: (-0.031213, 0.031270) T-Test of mean difference = 0 (vs not = 0): T-Value = 0.00 P-Value = 0.999					
Backorder Rate Part 2		N	Mean	StDev	SE Mean
	NSB2	45	0.136625	0.126965	0.018927
	B2	45	0.136602	0.126965	0.018927
	Difference	45	0.000023	0.165432	0.024661
95% CI for mean difference: (-0.049678, 0.049724) T-Test of mean difference = 0 (vs not = 0): T-Value = 0.00 P-Value = 0.999					

From the foregoing it seems that we have developed a method that poses high potential as an approximation method that will enable researchers to explore larger incarnations of the unidirectional substitution model with Markov Chains.

Summary of Unidirectional Substitution Model

Utilizing a Markov chain whose transition matrix is constructed using cumulative binomial probabilities, we have been able to successfully construct a unidirectional substitution model in which the reliability of a substitute part is different from the reliability of the primary part. In addition, we are able to successfully model substitution when the policy maker chooses to substitute some of the time but not all of the time.

We demonstrate that substitution always increases fill rate, and reduces the backorder rate, on the machine upon which the substitution is made (machine type 1). We also observe that the partial substitution scenario has great value to the policy planner because substitution is not a panacea for maximizing the customer satisfaction to cost ratio. In particular, substitution consistently improves the measures of goodness with respect to Part 1 inventory, machine 1 part fill rate, and machine 1 part backorder rate. However, this improvement is at the expense of decreased customer service, increased ordering, and increased steady state inventory on the machine whose primary part is used as a substitute (machine type 2).

Substitution reallocates machine type 2 inventory to meet machine type 1 demand. The policy maker must either compensate for this reallocation by raising Part 2 target stock levels or else accept lower machine type 2 part fill rate and higher machine type 2 backorder levels. Also, Part 2 ordering will rise with substitution and any cost increases from this must be considered when choosing substitution. Generally, the deterioration in machine type 2 performance metrics is less intense in magnitude than the increase in machine type 1 performance metrics. As a result, when the policy making decision utilizes minimum customer service indicators, overall backorder rate and fill rate improve with increasing substitution rates. Total inventory and ordering rise with increased substitution if Part 2 is less reliable on machine type 1 than the primary part and decrease when the opposite is true.

The tradeoff between inventory levels and customer service, as well as the tradeoffs between machine type 2 performance indicators and machine type 1 performance indicators, grows as the substitute becomes less reliable. The impact of reliability on the substitution results very much justifies the computational effort required to model dissimilar reliability.

Ultimately this model confirms the need for multidimensional analysis in order to execute effective decision making on service part substitution policy. With a good model for steady state performance indicators, and a solid understanding of the complex tradeoffs involved with substitution, the policy maker can analyze the cost ratio of the parts involved, the holding cost of the various inventories, and backorder costs associated with each machine class, and optimize expected value by matching the cost numbers with trade off relationships.

CHAPTER 5
GENERAL SUBSTITUTION OF A NON-PRIMARY PART
FOR MULTIPLE MACHINE TYPES

Introduction

In this section we examine the substitution problem from a different perspective and with an alternate approach than we have employed in the unidirectional substitution problem. The opportunity we explore in this case is one in which a general purpose or higher rank component is used as a substitute for the primary part for multiple machine types. The primary point of difference between this problem and the unidirectional substitution problem is the fact that the substitute is not a primary part for any of the machine types.

This type of substitution is ubiquitous in everyday life and also is common industry practice. A simple example of this type of substitution occurs when we place a spare tire on our car in place of the primary tire which has gone flat. We would never choose to operate with the spare tire as it gives a far less comfortable ride. However, the spare tire will get us home and prevents our machine from being non-operational.

Similarly, it is very common for service personnel to make a substitution of a part with some part not primary to any of the machines in use if that part might

keep a machine operational. What is not common is planning for the systematic use of such parts.

An example of this type of substitution in the pollution control industry follows. Imagine there are two processes, process one has a waste stream where there is only one contaminant and that contaminant is hydrogen fluoride. This type of contaminant can be scrubbed utilizing an adsorbent laden with ferric oxide. Process two has a waste stream containing arsine which is scrubbed very effectively with an adsorbent laden with cupric oxide. Neither of these adsorbents would work for the waste stream of the other process, but each is very effective and very safe on its targeted waste stream. However, it is possible to utilize an adsorbent comprised of activated carbon laden with various metal oxides to abate both of these processes. The caveat is that the carbon is not preferred to the primary parts because there is a danger of fire if throughput grows too high, and as such, production must be throttled or additional cooling must be supplied to the pollution control device (typically in the form of nitrogen gas purging).

The general substitution model could potentially offer many of the benefits of the unidirectional substitution model but is quite clearly different and must be modeled as such. In the following pages we will examine such a model.

Nomenclature of the General Substitution Model

S_A = Target inventory of part A

S_B = Target inventory of part B

S_C = Target inventory of part C

I_t^A = Inventory of part A at time t

I_t^B = Inventory of part B at time t

I_t^C = Inventory of part C at time t

n_A = Number of machine type A

n_B = Number of machine type B

P_A = Probability of substitution for part A

P_B = Probability of substitution for part B

λ_A = Failure rate of part A

λ_B = Failure rate of part B

μ_A = Delivery rate of part A

μ_B = Delivery rate of part B

μ_C = Delivery rate of part C

i = A subscript for the inventory state of I_t^A

j = A subscript for the inventory state of I_t^B

k = A subscript for the inventory state of I_t^C

$P_{i,j,k}$ = Steady state probability of state where $I_t^A = i$ and $I_t^B = j$ and $I_t^C = k$

$Down_{i,j,k}$ = The rate of moving from state I_t^A, I_t^B, I_t^C to a state $(I_t^A - 1), I_t^B, I_t^C$

$Up_{i,j,k}$ = The rate of moving from state I_t^A, I_t^B, I_t^C to a state $(I_t^A + 1), I_t^B, I_t^C$

$Left_{i,j,k}$ = The rate of moving from state I_t^A, I_t^B, I_t^C to a state $I_t^A, (I_t^B + 1), I_t^C$

$Right_{i,j,k}$ = The rate of moving from state I_t^A, I_t^B, I_t^C to a state $I_t^A, (I_t^B - 1), I_t^C$

$In_{i,j,k}$ = The rate of moving from state I_t^A, I_t^B, I_t^C to a state $I_t^A, I_t^B, (I_t^C - 1)$

$Out_{i,j,k}$ = The rate of moving from state I_t^A, I_t^B, I_t^C to a state $I_t^A, I_t^B, (I_t^C + 1)$

General Substitution Model

In our model there are two machine types which we will refer to as machine type A and machine type B. Each of these machines uses a primary part which we refer to as Part A and Part B respectively. Part A and Part B are sufficiently different in order to create a scenario in which neither part could be used as a substitute for the other part. There is, however, another part that can be used as a substitute part for either Part A or Part B; we will refer to this part as Part C. Although Part C can be used as a substitute for either Part A or Part B, it is not as desirable to use Part C on either machine class as it is to use the primary component, and as a result the customer would prefer to use the primary component (consider our spare tire analogy). For example, Part C might be more expensive than either of the primary parts or it might have a higher storage or shipping cost or might simply be annoying (noisy, smelly, etc.).

In this scenario, part replacement and ordering occur on a continuous basis and when a part fails it is replaced with its primary part if that part is available. Parts are ordered such that a predetermined target inventory for each part type is met. In this ordering plan new orders are equal to the target inventory minus current inventory (unless an overstock scenario exists, in which case no orders are made). If the primary part is not in stock, the substitute part – Part C – might be used to maintain the operation of the machinery and if a choice

is made to use this part it is immediately placed into service. In our model, substitution is not always mandated under all policies and the probability that substitution will occur is a decision variable that is controlled by policy makers. If no inventory for a primary part is available upon the arrival of a failure, and substitution is not performed, then a backorder state exists and a machine is idled. The idled machine and the associated part backorder state are represented in our inventory model as a negative inventory.

We define the state of the system as the inventory level of Part A, Part B, and Part C. The state of the system is noted at any change in any inventory immediately upon the occurrence of the change; as a result, transient states are represented although they would immediately be rectified by service personnel to correct the suboptimal configuration represented. As an example, we might have the state (I_t^A, I_t^B, I_t^C) where I_t^A is negative and I_t^C is positive even though the substitution policy is $PS = 1$. Service personnel would immediately perform a substitution that raised I_t^A by 1 and lowered I_t^C by 1; however, this substitution would not change the fact that the state (I_t^A, I_t^B, I_t^C) had existed and hence there is some steady state probability associated with being in state (I_t^A, I_t^B, I_t^C) , exiting state (I_t^A, I_t^B, I_t^C) , and entering state (I_t^A, I_t^B, I_t^C) . As a result, the rate of entry and exit to the transient state is a positive value as is the steady state probability of state (I_t^A, I_t^B, I_t^C) .

We assume (1) that the quantity of each type of machine is sufficiently large and of a sort that the failure rate of each part type is well known and that

the arrival rate of failures follows a Poisson distribution. We further assume (2) that an idle condition on any machine does not impact the failure rate perceptibly.

The first assumption (1) is fairly straightforward and is common in the reliability literature. It is based upon the principle that by accumulating a large number of non-Poisson rare events that have no natural tendency to cluster, or Poisson processes yield a Poisson Process (Liu and Lee, 2007). Alternatively, this assumption could be realized if the part itself exhibited exponential failures as an innate characteristic (such as is the case with many electronic components). The ultimate verification of this result is empirical as we assume that we observe Poisson arrivals with mean λ . The second arrival assumption (2) is based upon observation and the presence of a sufficiently large number of machines. In this case we have a sufficiently large population of machines where the potential for any given arrival is rare and is not influenced by prior failures. Further, we assume that the service rate for new part delivery is exponential. In addition to the foregoing we assume exponential service rates, also common in the reliability literature.

Changes in state with respect to parts arrivals are a function of the difference between inventory level and inventory set point multiplied by the rate of arrival for a given part. The parts arrival rate rules are presented for visualization as movement along the appropriate axis in a three dimensional chart with dimensions $(H \times W \times D) = [(S_A + n_A + 1) \times ((S_B + n_B + 1) \times (S_C + 1))]$. The movement rules for parts arrivals are:

$$Up_{i,j,k} = (S_A - I_t^A) \mu_A \quad (5.1)$$

$$Left_{i,j,k} = (S_B - I_t^B) \mu_B \quad (5.2)$$

$$Out_{i,j,k} = (S_C - I_t^C) \mu_C \quad (5.3)$$

Movement with respect to failures must consider the fact that substitution will impact the direction of movement. Probability of substitution is dictated by limits imposed by inventory in that substitution does not occur if adequate stock of the primary part exists, nor does it occur if inadequate quantities of the substitute are available. In addition the probability of substitution is further constrained in this model by the fact that we allow the policy maker to set the substitution rate as a decision variable. Probability of substitution is subject to the following rules:

$$P_A = \begin{cases} 0 & \text{for } I_t^A, I_t^B, I_t^C \text{ such that } I_t^A > 0 \cup I_t^C = 0 \\ \text{As defined otherwise} \end{cases} \quad (5.4)$$

$$P_B = \begin{cases} 0 & \text{for } I_t^A, I_t^B, I_t^C \text{ such that } I_t^B > 0 \cup I_t^C = 0 \\ \text{As defined otherwise} \end{cases} \quad (5.5)$$

Now that we have defined the rules governing the probability of substitution we can address movement resulting from part failures. This movement is governed by the following relationships.

$$\text{Down}_{i,j,k} = \begin{cases} 0 & \text{if } I_t^A = -n_A \\ \lambda_A & \text{if } I_t^A > 0 \text{ or } (I_t^C = 0 \text{ and } 0 \geq I_t^A > -n_A) \\ \lambda_A (1 - P_A) & \end{cases} \quad (5.6)$$

$$\text{Right}_{i,j,k} = \begin{cases} 0 & \text{if } I_t^B = -n_B \\ \lambda_B & \text{if } I_t^B > 0 \text{ or } (I_t^C = 0 \text{ and } 0 \geq I_t^B > -n_B) \\ \lambda_B (1 - P_B) & \end{cases} \quad (5.7)$$

$$\text{In}_{i,j,k} = \begin{cases} \lambda_A P_A & \text{if } I_t^A \leq 0 \text{ and } I_t^B > 0 \text{ and } I_t^C > 0 \\ \lambda_B P_B & \text{if } I_t^B \leq 0 \text{ and } I_t^A > 0 \text{ and } I_t^C > 0 \\ \lambda_A P_A + \lambda_B P_B & \text{if } I_t^A \leq 0 \text{ and } I_t^B \leq 0 \text{ and } I_t^C > 0 \\ 0 & \text{otherwise} \end{cases} \quad (5.8)$$

The preceding relationships in equations 5.1 through 5.8 collectively govern the rate of movement into each state set in the state set universe. They also govern the rate of movement out of each state set in the state set universe. At steady state we cannot have accumulation and the rate of movement into each state must equal the rate of movement out of that state.

It necessarily follows that equation 5.9 must hold for the entire network of cells in order to maintain steady state.

$$\begin{aligned} P_{i,j,k} (\text{Up}_{i,j,k} + \text{Left}_{i,j,k} + \text{Down}_{i,j,k} + \text{Right}_{i,j,k} + \text{Out}_{i,j,k} + \text{In}_{i,j,k}) & \quad (5.9) \\ = P_{(i-1),j,k} \text{Up}_{(i-1),j,k} + P_{i,(j-1),k} \text{Left}_{i,(j-1),k} & \\ + P_{(i+1),j,k} \text{Down}_{(i+1),j,k} + P_{i,(j+1),k} \text{Right}_{i,(j+1),k} & \\ + P_{i,j,(k+1)} \text{In}_{i,j,(k+1)} + P_{i,j,(k-1)} \text{Out}_{i,j,(k-1)} & \end{aligned}$$

$$\forall \quad -n_A \leq I_t^A \leq S_A \quad -n_B \leq I_t^B \leq S_B \quad 0 \leq I_t^C \leq S_C$$

The steady state requirement can be combined with the requirement that, since we are in exactly one state at any given point in time, all probabilities must sum to one in order to generate a set of equations sufficient to produce unique steady state probabilities for each state set.

$$\sum_{i,j,k} P_{i,j,k} = 1 \quad (5.10)$$

For:

$$\forall \quad -n_A \leq I_t^A \leq S_A \quad -n_B \leq I_t^B \leq S_B \quad 0 \leq I_t^C \leq S_C$$

Figure 5.1 shows a graphical representation of the state transition model. Movement in the inventory of Part A is on the vertical axis, movement of Part B state is on the horizontal axis, and movement with respect to substitute inventory (Part C) is upon that axis which traverses in and out of the paper.

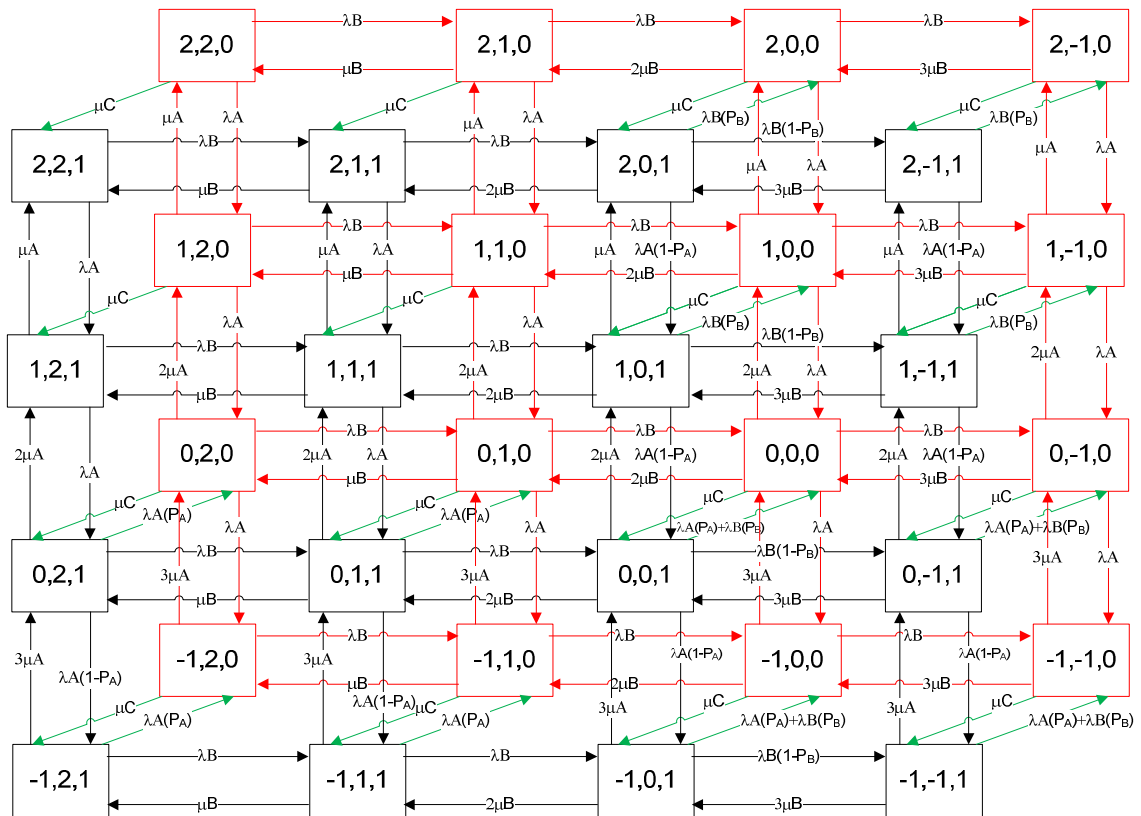


Figure 5.1 Visual Representation of the State Transition Model

Implementation of the Algorithm

The algorithm described above was solved in Mathematica 7.0 utilizing the programming characteristics of that software to create a conditional looping structure to generate the state set and movement rules. We then use the solving capabilities of that Mathematica to solve the resultant system of linear equations. The algorithm is in the appendix at the end of this work for interested readers to examine.

For all models considered, we examined the scenario where the number of machines of each class was 30; i.e. $n_1 = n_2 = 30$. We varied S_A and S_B independently from a minimum value of 3 to a maximum value of 8. We varied S_C from 0 to 3 and P_A and P_B through the range $\{0, 0.25, 0.5, 0.5, 0.75, 1.0\}$. For the case in which P_S was zero, S_C was held strictly at zero because there is no policy where it makes sense to stock Part C when there is no possibility of substitution. Although the actual size of the state set varied depending upon the values tested for S_A , S_B , and S_C , the minimum number of unique entities in the state sets was 1024 for $S_A=S_B=1$ and $S_C = 0$ (no substitution model) while the maximum span was 6084 for $S_A = S_B = 8$, $S_C = 3$ (maximum target inventories studied). Of course, as was shown in Equation 5.9, this gave us 6084 equations in the same number of variables (with equation 5.10 incorporated into the system of equations, one of the equations generated by equation 5.9 was discarded so that the number of equations had the proper degrees of freedom).

In our algorithm, we assigned different values to the reliability of the individual parts with $\lambda_A = 0.3$ and $\lambda_B = 0.6$. We assigned the same arrival rate for

parts to each part type, thereby yielding two separate scenarios for the relationship between a given part class and its arrival rate, with $\mu_A = \mu_B = 0.1$. We varied μ_C through the sequence { 0.2, 0.1, 0.05 }, thereby creating the opportunity to test for the sensitivity of substitution measures of goodness with service rates on the substitute half the value of a primary part, equal to a primary part, and double that of a primary part.

Analysis of Data from General Model

Since customer satisfaction is often a motivation for substitution, and is always a goal of the successful enterprise, we begin our analysis by examining a scenario where we target a minimum total fill rate; where fill rate is defined as that portion of the time when we satisfy all demand.

In this scenario, we examine the case where $\mu_C = 0.5$ (substitute can be replenished half as fast as either primary part) across the array of substitution policies for those scenarios in which a total fill rate of at least 90% is demonstrated.

Table 5.1 shows those inventory policies that result in fill rates above the targeted minimum level of 90%. Several things are apparent from examining these results. First, higher probability of substitution clearly leads to higher fill rates. This is expected due to the fact that foregoing substitution when the opportunity is presented is equivalent to electing to leave a machine down. We might well ask the question; “what is the best inventory policy for these substitution rates?” In answer, the first three lines of the table are very revealing

because we can observe that a very small drop in total fill rate results in significantly reduced inventory for Part A and a slight decrease in Part C inventory. Consideration should certainly be given to a policy where we can lower inventory by 10.5% while dropping fill rate by only 1.2% and still achieve a fill rate significantly above the minimum target level.

Another very interesting aspect of the results from our model can be observed in Table 5.1 near the bottom of the results. If we compare the 17th and 18th rows we note that a substitution policy where substitution occurs only 75% of the time appears superior to a policy where substitution is always performed. The decision variable combination with $S_A = 5$, $S_B = 8$, $S_C = 3$ with $P_s = 0.75$ has slightly higher inventory for Part B and Part C than the $S_A = 8$, $S_B = 7$, $S_C = 3$ with $P_s = 1.0$ policy. However, the former policy has a significantly reduced inventory of Part A and a slightly higher total fill rate. Note that the (5, 8, 3, 0.75) policy has a total steady state inventory of 6.51 vs. the sum of inventory for the (8, 7, 3, 1) policy which has a total steady state inventory of 8.45. Of course, consideration would have to be given to the relative cost of Part A vs. Part B, but it is very likely that the policy incorporating the lower probability of substitution is superior in this instance. Many similar examples can be found in this table and in other sections of the data. The key message is that, although substitution can greatly enhance fill rate at any given inventory level, care must still be taken to optimize substitution policy for the target inventory for each part class. Moreover, if one product class is significantly more expensive to carry in inventory then

substitution can be used to reduce the inventory of that part while maintaining customer satisfaction.

Table 5.1 Inventory for Policies with Fill Rate Greater than 90 Percent

Total Fill Rate	Probability of Substitution	S _A	S _B	S _C	Part A Inventory	Part B Inventory	Part C Inventory
95.46%	1	8	8	3	5.02	2.61	1.91
95.14%	1	7	8	3	4.05	2.60	1.86
94.29%	1	6	8	3	3.13	2.60	1.76
94.28%	0.75	8	8	3	5.02	2.58	2.00
93.91%	0.75	7	8	3	4.05	2.58	1.96
92.95%	0.75	6	8	3	3.12	2.57	1.87
92.80%	1	8	8	2	5.02	2.54	1.17
92.55%	0.5	8	8	3	5.01	2.53	2.15
92.36%	1	7	8	2	4.05	2.53	1.13
92.26%	1	5	8	3	2.27	2.58	1.59
92.12%	0.5	7	8	3	4.04	2.53	2.11
92.04%	0.75	8	8	2	5.02	2.52	1.22
91.58%	0.75	7	8	2	4.04	2.51	1.19
91.22%	1	6	8	2	3.11	2.52	1.06
90.99%	0.5	6	8	3	3.10	2.53	2.03
90.91%	0.5	8	8	2	5.01	2.49	1.32
90.65%	0.75	5	8	3	2.25	2.56	1.70
90.42%	1	8	7	3	5.02	1.91	1.52
90.41%	0.5	7	8	2	4.04	2.49	1.29
90.36%	0.75	6	8	2	3.10	2.51	1.13

Table 5.2 demonstrates a targeted backorder policy with backorders held to less than one for each machine type part. There is clearly decreased inventory for equivalent backorder levels as the probability of substitution rises (see highlighted region).

Table 5.2 Policies where Backorder Rate for Both Parts is Below 1

P_s	S_A	S_B	S_C	I_{SS}^A	I_{SS}^B	I_{SS}^C	Total Inventory	Part A Backorders	Part B Backorders
0%	3	6	0	0.67	0.96	0	1.64	0.67	0.96
100%	3	6	1	0.73	1.05	0.16	1.93	0.55	0.8
75%	3	6	1	0.72	1.04	0.19	1.95	0.56	0.8
50%	3	6	1	0.72	1.04	0.23	1.99	0.57	0.81
100%	3	5	3	0.8	0.73	0.5	2.03	0.4	0.88
25%	3	6	1	0.71	1.03	0.34	2.08	0.58	0.82
75%	3	5	3	0.79	0.72	0.6	2.1	0.41	0.9
0%	3	7	0	0.67	1.57	0	2.24	0.67	0.57
50%	3	5	3	0.77	0.69	0.78	2.25	0.43	0.93
100%	3	6	2	0.78	1.13	0.38	2.28	0.44	0.65

As we indicated earlier, one area we wish to explore is whether a differential in delivery time could impact the benefits of substitution. It is often the case that a part can be garnered locally or with a fast lead and it may be tempting to consider such parts as substitutes in order to enhance customer satisfaction or maintain smaller inventory for a given level of customer satisfaction.

The plot in Figure 5.2 explores the relationship between the policy variables, target inventory levels and substitution policy, vs. the dependent measure of goodness, total fill rate. This plot demonstrates how the relationship

varies as the substitute part becomes ever more available. The bottom layer in the plot shows a substitute that arrives half as rapidly as a standard part ($\mu_C = 0.5$), the middle layer shows the results when the substitute arrives at the same rate as the primary parts ($\mu_C = 1.0$), and the top layer shows the results when the substitute part arrives twice as fast as the primary parts ($\mu_C = 2.0$).

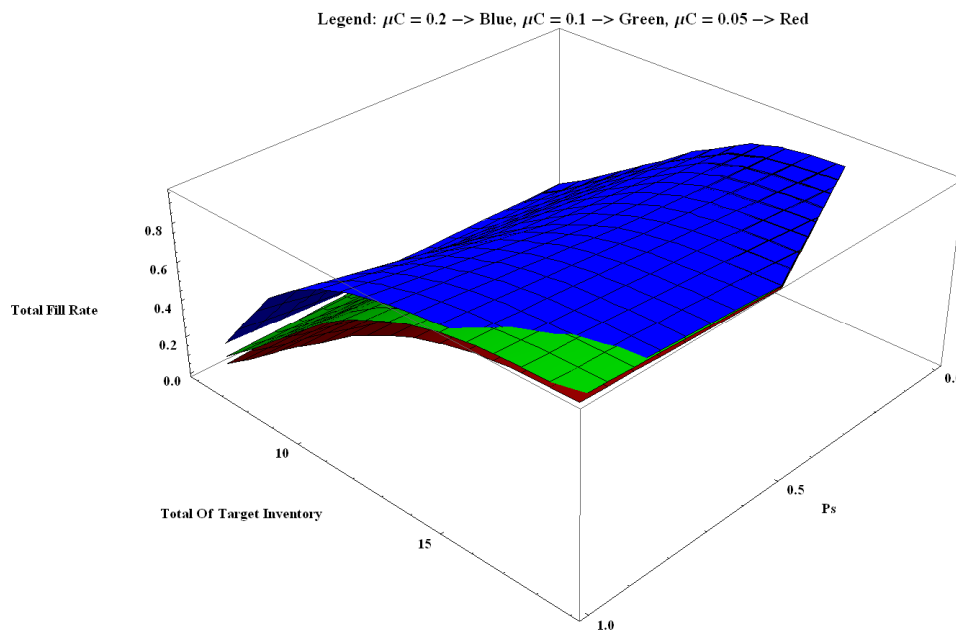


Figure 5.2 Total Fill Rate as a Function of Total Target Stock and the Probability of Substitution at Various Substitute Service Rates

Next we examine the tradeoff between inventory levels and customer satisfaction; in this case, customer satisfaction as manifested by backorder rate. We analyze a case where we lock S_C at 3 units so that we can assume substitute availability is present if desired. We set the arrival rate of the substitute μ_C , at 0.05 (half the rate of the arrival rate of either μ_A , or μ_B -- the primary parts) and observe that, even with longer lead times for the substitute, substitution still

results in either lower inventory for a given customer service level, or a higher customer service level is achieved from a given inventory position with substitution. Figure 5.3 is a graphical representation of a comparison between the backorder rates associated with various inventory positions for a no-substitution policy ($P_A = P_B = 0$) and for an always-substitute policy ($P_A = P_B = 1$). The cones represent the no-substitution results and the cubes represent the substitute-always results. The thickness of the base of both the cones and the cubes represents the inventory level associated with that data point. The height of the shape represents the backorder rate. What is apparent from this figure is that, for corresponding inventory positions, the no-substitution policy consistently results in significantly higher backorder rates. Also apparent in the figure is the fact that those shapes with thick bases consistently have lower backorder rates than those with narrow bases. In essence, this data confirms the fact that substitution can result in a lower backorder rate at any given inventory position. The data also confirms the corollary to the former statement; that is, a given level of customer service (given backorder rate) can be achieved with a lower inventory position if substitution is allowed than if substitution is not allowed.

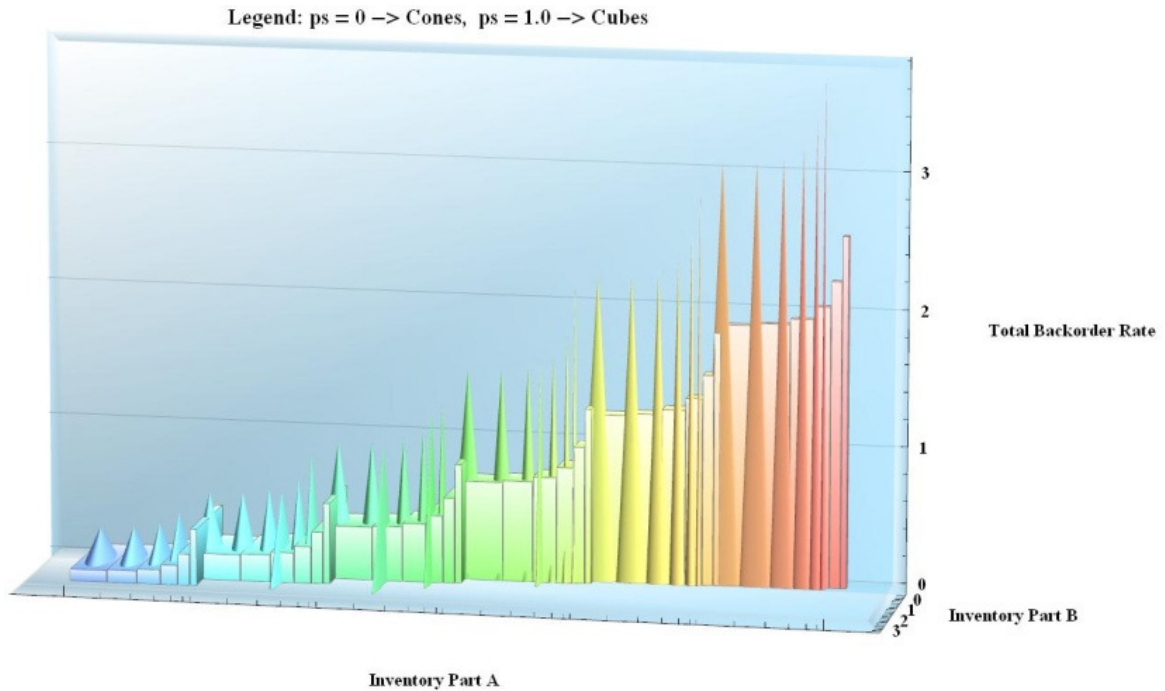


Figure 5.3 Weighted Bar Chart Showing the Backorder Rate Associated with an Array of Inventory Positions for $\mu_C = 0.05$

As a further step in this experiment we examine how this relationship is altered if the substitute is more readily available than the primary parts. This is a very realistic scenario as substitute parts are frequently those that can be found in the local market or are more generally manufactured. Figure 5.4 shows the results of this experiment with all of the inputs the same as was the case in the preceding trial excepting the fact μ_C was 0.20 in this trial. Under these conditions, the superiority of high substitution policies, with respect to backorder rates at given inventory levels, becomes much more pronounced than was the case when substitute parts were longer lead. The message is that if substitute parts are readily available on shorter lead times than the primary parts then the

case for substitution is much stronger with respect to a customer service vs. inventory model.

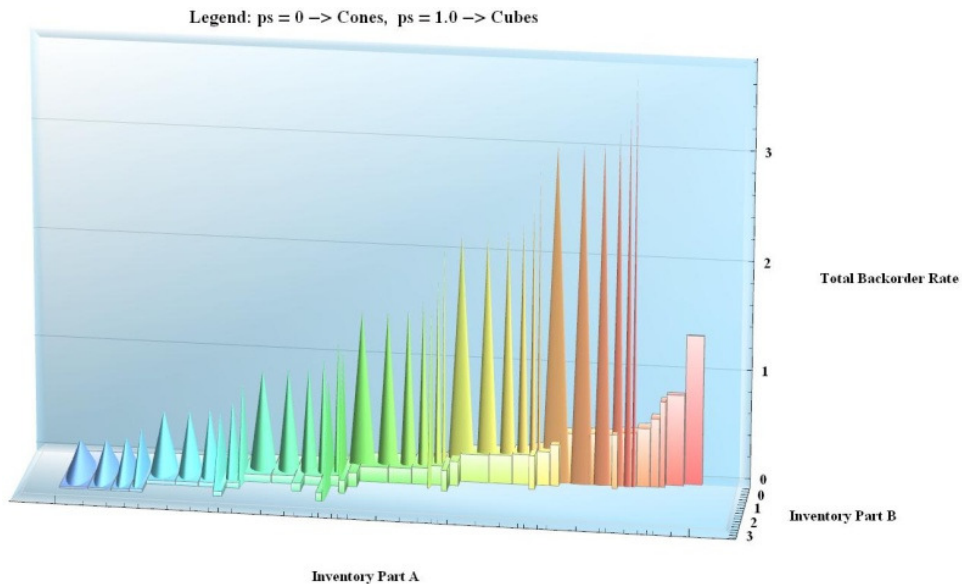


Figure 5.4 Weighted Bar Chart Showing the Backorder Rate Associated with an Array of Inventory Positions for $\mu_C = 0.20$

We further investigate the interplay between inventory levels and substitution by examining the partial-substitution policy where $P_A = P_B = 0.5$ and find, not surprisingly, that the same dynamics with respect to inventory levels vs. backorder rates happen when a partial substitution policy is employed. We present the weighted bar chart in Figure 5.5 which shows that the results for the partial substitution policy are not as desirable as the full-substitution policy, but are nonetheless more desirable than the no-substitution policy with respect to backorder vs. inventory.

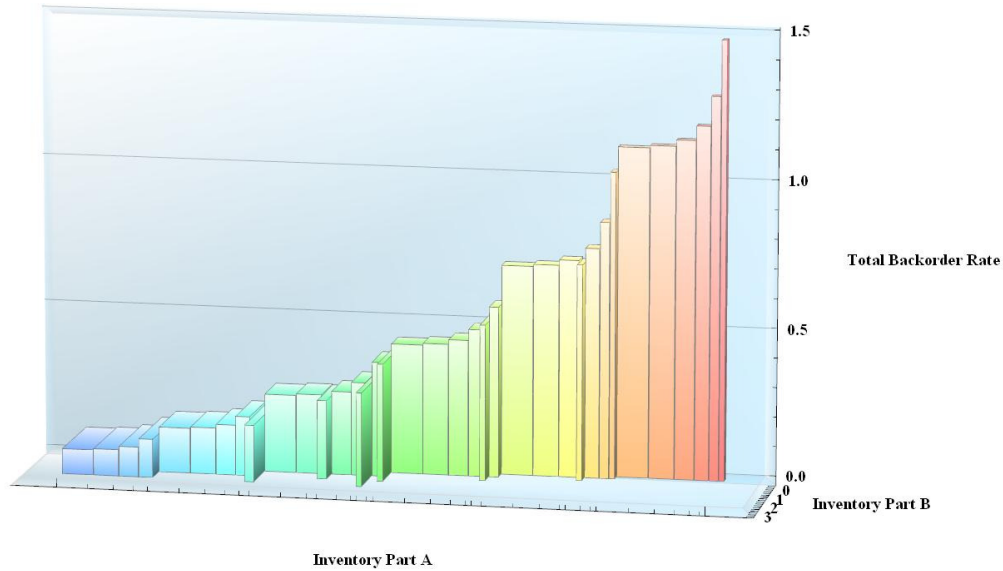


Figure 5.5 Weighted Bar Chart Showing the Backorder Rate Associated with an Array of Inventory Positions for $\mu_C = 0.20$ and a partial substitution policy

As a follow up to the previous line of inquiry, we next investigate the scenario where specific customer service levels are targeted and then from that point the objective is to minimize cost at those service levels. In this case we constrained the policy to include those cases where the backorder rate for Part A and the backorder rate for Part B were between 0.05 and 0.25. The upper boundary was designed to maintain high overall uptime on the equipment, while the lower boundary was used to ensure that the expense resulting from setting unnecessarily high targets was not incurred. In actual practice, these numbers will be very specific to the application and the customer. Lower backorder rates would be associated with life safety and high value processes while less critical operations might target higher backorder values and lower operating cost.

Table 5.3 shows those cases that fall within the backorder range arrayed from lowest total inventory to highest total inventory. Although all of the values fall within the acceptable service range when service is defined by the backorder rate, it is clear that higher inventory levels are associated with lower backorder rates. The same could not necessarily be said for fill rates as it is clear that fill rate is more complicated and relies upon a combination of the inventory of the part associated with that fill rate and with the number of substitutes available.

Although generally it is true that higher substitution levels correlate with lower backorder levels at a given inventory point, closer inspection of the results indicates that more analysis needs to be done in order to develop a policy. As evidence of the foregoing, we submit a comparison of the two highlighted rows in Table 5.3. We will refer to each of these two rows with the decision variable policy (S_A, S_B, S_C, P_S) : note that $P_S = P_A = P_B$ in this case. If we compare $(5,8,1,0.25)$ with $(4,8,3,1)$ we first note that the two policies have essentially identical backorder rates for Part A and fill rates for Part A. Policy $(5,8,1,0.25)$ has a lower total inventory than $(4,8,3,1)$, however the cost of the lower inventory is a significantly poorer backorder rate for Part B and a poorer fill rate for Part B.

An analysis of the comparison between policy $(5,8,1,0.25)$ and policy $(4,8,3,1)$ would have to weigh any cost savings from lower inventory against the decreased customer service costs. Since both policies meet the backorder target, both are acceptable, and the lower inventory of policy $(5,8,1,0.25)$ might become attractive. However, note the lower inventory is a total inventory position; the individual components of the inventory position are quite different

with policy (5,8,1,0.25) having a higher inventory of Part A but lower inventories of Part B and Part C than policy (4,8,3,1). As such, the relative costs of Part A, Part B, and Part C must be considered before simply using total inventory as a criterion for cost minimization.

Table 5.3 Inventory Rate for Part A and Part B Backorder Level Less than 1.0

S _A	S _B	S _C	Probability of Substitution	Inventory Part A	Inventory Part B	Inventory Part C	Total Inventory	Part A Backorder Rate	Part B Backorder Rate	Part A Fill Rate	Part B Fill Rate
4	8	1	1	1.396	2.405	0.333	4.134	0.229	0.238	0.862	0.880
4	8	1	0.75	1.389	2.400	0.368	4.158	0.234	0.239	0.858	0.879
4	8	1	0.5	1.380	2.393	0.421	4.194	0.241	0.242	0.853	0.876
4	7	3	1	1.488	1.849	1.079	4.415	0.128	0.247	0.919	0.876
4	8	2	1	1.459	2.485	0.775	4.720	0.159	0.173	0.902	0.910
4	8	2	0.75	1.445	2.474	0.852	4.771	0.163	0.177	0.893	0.906
4	8	2	0.5	1.425	2.457	0.979	4.880	0.185	0.185	0.880	0.899
5	8	1	1	2.192	2.420	0.413	5.025	0.092	0.226	0.941	0.886
5	8	1	0.75	2.187	2.414	0.445	5.045	0.095	0.229	0.939	0.884
4	8	2	0.25	1.391	2.421	1.235	5.048	0.221	0.209	0.860	0.886
5	8	1	0.5	2.180	2.404	0.496	5.080	0.099	0.233	0.935	0.890
5	8	1	0.25	2.167	2.384	0.601	5.153	0.107	0.246	0.930	0.873
4	8	3	1	1.509	2.554	1.329	5.392	0.106	0.122	0.932	0.935
4	8	3	0.75	1.487	2.535	1.458	5.477	0.121	0.128	0.919	0.928
5	7	3	0.75	2.240	1.851	1.393	5.484	0.055	0.232	0.961	0.877
4	8	3	0.5	1.455	2.505	1.684	5.623	0.148	0.143	0.899	0.917
5	8	2	1	2.237	2.509	0.943	5.689	0.061	0.157	0.960	0.919
5	8	2	0.75	2.226	2.494	1.015	5.735	0.066	0.162	0.955	0.913
5	8	2	0.5	2.210	2.471	1.132	5.814	0.075	0.173	0.949	0.905
4	8	3	0.25	1.405	2.447	2.067	5.919	0.202	0.184	0.868	0.896
5	8	2	0.25	2.185	2.428	1.362	5.975	0.092	0.203	0.938	0.889
5	8	3	0.5	2.229	2.518	1.889	6.636	0.060	0.132	0.957	0.922
5	8	3	0.25	2.194	2.452	2.236	6.882	0.084	0.179	0.941	0.898

Summary of General Substitution Model

The research on a substitution model that employs a part held solely for the purpose of acting as a substitute for primary parts (which we refer to as a general substitution model) demonstrates that substitution policies that maintain inventory solely for the purpose of substitution can have significant positive impacts on customer service levels and inventory holding costs.

In this research we develop a model utilizing a Poisson process and solve the equation set for the ternary part model for steady state probabilities and performance indicators. The results clearly indicate that this method can have a significant and positive impact on customer performance metrics. This model requires the coordination of additional inventory components and this factor may cause some issues when inventory is seriously constrained. However, unlike unidirectional substitution, this method does not cause disruption to any primary part performance measure, as the primary parts are not reallocated to alternate demands. The results from this model demonstrate that the general substitution strategy can increase part demand satisfaction while reducing overall inventory.

Modeling of probabilistic substitution policies demonstrates that an all-or-nothing approach to substitution is often not optimal. At sub-maximum targeted performance levels a partial substitution strategy can yield acceptable results with significantly decreased inventory. Ultimately the policy must be set by weighing the cost of the respective inventories against backorder costs while meeting part demand lead targets.

CHAPTER 6

CONCLUSION

Concluding Remarks

This research demonstrated that substitution adds increased flexibility to service parts inventory policy planning and offers the opportunity to improve the customer-performance-to-cost ratio. We have succeeded in completing the tasks that we earlier indicated presented an opportunity to add to the literature of the profession.

In the unidirectional substitution model we were able to reach beyond myopic models and single-period models to develop steady state performance indicators for a reasonably large sized problem. In addition, we were able to model substitution with replacement parts whose reliability was not the same as the reliability of the primary part. Since a substitute part will seldom have the same reliability on a machine for which it is not the primary part as it does on its native application, the completion of this model helps to bring the substitution literature closer to the actual behavior of these parts in commercial applications.

We also presented a steady state model to examine long-term behavior of a substitution policy in which a part that was not a primary part for any system was carried in inventory solely for the purpose of substitution. This model addressed a real-world practice that is not addressed in the service parts

literature. The model presented in this dissertation is able to project the long-term ramifications of substitution decisions. This model demonstrated that the use of parts solely as substitutes can be a powerful tool in increasing customer service levels without an accompanying increase in inventory levels.

An approximation method with a high degree of conformity to non-approximated results was presented and validated through statistical testing. This method, the Boycott Method, will allow the examination of considerably larger substitution problems within the constraints provided by modern computing systems.

All of the models that we examined allowed for a partial substitution strategy, thus facilitating policy planning that optimizes the level of substitution so that policy makers can fine tune the tradeoffs between inventory and ordering cost increases against improved customer service.

In all cases, the models do not present a simple cookbook solution to determine optimal parts management strategy. Rather, the models provide information on long term behavior so that strategic planners can view the change in performance indicators within the context of their unique inventory costs and constraints, variable part cost ratios, and customer service requirements. Utilizing their specialized knowledge of the operation, in conjunction with the tools presented in this dissertation, strategic planners should be able to increase their ability to achieve higher customer service levels while maintaining, or even reducing, operating costs.

Areas for Further Research

The unidirectional substitution model considered single-day and two-day lead times and assumed that lead time was known with certainty. Further research upon the impact of increased lead time and upon the impact of uncertainty in lead time could lead to increased conformity with a larger number of applications in industry.

Adapting the general substitution model to a Markov chain would provide the ability to gather more information on variable costs and facilitate the direct comparison of the unidirectional substitution model with the general purpose substitution model. A Markov chain solution algorithm for the general purpose substitution problem would also permit the use of dissimilar reliability using the methodology put forth in this dissertation for the unidirectional substitution model.

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APPENDIX A
MATHEMATICA CODE FOR THE SOLUTION OF THE
UNIDIRECTIONAL SUBSTITUTION MODEL

We will use the following set of definitions

First we declare the values for the key parameters of the model $n1$ = number of machine type 1

$n2$ = number of machine type 2

$s1$ = stock to level for part 1

$s2$ = stock to level for part 2

$s1_{max}$ = iterator limit for $s1$

$s2_{max}$ = iterator limit for $s2$

i = inventory position of Part 1

j = inventory position of Part 2

k = number of substitutions active

$v1$ = number of part type 1 on order to be delivered + 1 days

$v2$ = number of part type 1 on order to be delivered + 2 days

$w1$ = number of part type 2 on order to be delivered + 1 days

$w2$ = number of part type 2 on order to be delivered + 2 days

$m1$ = number of failures of Part 1 on machine type 1

$m2$ = number of failures of Part 2 on machine type 2

$m21$ = number of failures of Part 2 on machine type 1

$m1_{max}$ = maximum number of possible failures of Part 1 on machine type 1

$m2_{max}$ = maximum number of possible failures of Part 2 on machine type 2

$m21_{max}$ = maximum number of possible failures of Part 2 on machine type 1

$q1$ = maximum number of binomial trials Part 1 on machine type 1

$q2$ = maximum number of binomial trials Part 2 on machine type 2

$q21$ = maximum number of binomial trials Part 2 on machine type 1

$basestates$ = the array that holds the list of valid states

$p1$ = probability that part 1 will fail

$p2$ = probability that part 2 will fail on machine type 2

$p21$ = probability that part 2 will fail on machine type 1

ps = probability that a substitution will be made given the opportunity to substitute

$maxn21$ = maximum number of possible substitutions

$nX21$ = maximum number of substitutions for specific transition

$n21$ = actual number of substitutions

$I1$ = Average inventory of Part 1

$I2$ = Average inventory of Part 2

Δ = Average level of substitution

$E1$ = Average excursions of Part 1

$E2$ = Average excursions of Part 2

$E12$ = Average excursions of either part

$F1$ = Fillrate of Part 1

$F2$ = Fillrate of Part 2

$F12$ = Fillrate of all parts

$B1$ = Backorder rate Part 1

$B2$ = Backorder rate Part 2

```

kount1, kount2, kount3, kount4, kount5 flag, a, e, x, y = counters
(*First some housecleaning. Clear all values*)
ClearAll["Global`*"]; starttime=DateString[]; Print[AbsoluteTime[starttime]];
(*We give the extents of the setpoint for this trial*)
s1max=5; s2max=5;
(*We give the number of machines for this trial*)
n1=10; n2=10;
(*We declare the probability of failure for each part use*)
p1 = .1; p2 = .1;
(*We set fixed boundaries*)
imin=-n1; jmin=-n2; kmin=0; v1min=0; v2min=0; w1min=0; w2min=0;
(*We establish the range for maximum failures*)
m1max=Compile[{{i,_Integer},{k,_Integer}},n1-k+Min[0,i]];
m2max=Compile[{{j,_Integer}},n2+Min[0,j]];
m21max =Compile[{{k,_Integer}},k];

binomial=Compile[{{K,_Integer},{N,_Integer},{p,_Real}},Which[N==0&&K==0,1,N==0&&K≠0,0,N≠0,PDF[BinomialDistribution[N,p],K]];
maxn21=Compile[{{i,_Integer},{j,_Integer},{v1,_Integer},{w1,_Integer},{m1,_Integer},{m2,_Integer},{m21,_Integer},{ps,_Real}},
If[i+v1-m1-m21≥0||j+w1-m2≤0||ps==0,0,Min[j+w1-m2,(m1+m21)-(i+v1)]]];
(*Innerflag is a lower limit for looping that moves as a function of state variable conditions*)
innerflag=Compile[{{i,_Integer},{j,_Integer},{v1,_Integer},{w1,_Integer},{m1,_Integer},{m2,_Integer},{m21,_Integer},{ps,_Real}},If[ps<1,0,If[i+v1-m1-m21≥0||j+w1-m2≤0||ps==0,0,Min[j+w1-m2,(m1+m21)-(i+v1)]]]];
(*We populate the column headers for the output chart*)
results=
({
{"I1", "I2", "Δ", "V1", "W1", "F1", "F2", "F12", "B1", "B2", "s1", "s2", "ps", "p21"}
});

(*grandBinomial is the cumulative probability of a state transition*)
grandBinomial=Compile[{{K1,_Integer},{N1,_Integer},{K2,_Integer},{N2,_Integer},{K3,_Integer},{N3,_Integer},{K4,_Integer},{N4,_Integer},{prs,_Real}},(Which[N1==0&&K1==0,1,N1==0&&K1≠0,0,N1≠0,PDF[BinomialDistribution[N1,p1],K1]])*(Which[N2==0&&K2==0,1,N2==0&&K2≠0,0,N2≠0,PDF[BinomialDistribution[N2,p2],K2]])*(Which[N3==0&&K3==0,1,N3==0&&K3≠0,0,N3≠0,PDF[BinomialDistribution[N3,p21],K3]])*(Which[N4==0&&K4==0,1,N4==0&&K4≠0,0,N4≠0,PDF[BinomialDistribution[N4,prs],K4]]]);
(*pValues are the probability of a given state transition*)
DistributeDefinitions[grandBinomial,maxn21,binomial];
3462661586

```

```

(*Here we enter the master loop that increments key model
inputs including inventory level*)
Do[
  Do[
    Do[

imax=s1+Min[s2,n1];jmax=s2;kmax=n1;v1max=n1;v2max=n1;w1max=n
2+s2;w2max=n2+s2;
(*Next we fill up the array of permissible states by
applying our state variable rules*)
If[ps<= 0.5||ps>= 0.75,
  Clear[locator];
  validstates={
    {s1, s2, 0, 0, 0}
  };
  locator[s1,s2,0,0,0]=1;
  target={
    {0, 0, 0, 0, 0}
  };
  basestates=validstates;
  For[flag=1,flag<=Length[basestates],flag++,
    For[m1=0,m1<=Min[n1-
basestates[[flag,3]]+Min[0,basestates[[flag,1]]],4],m1++,
    For
[m2=0,m2<=Min[n2+Min[0,basestates[[flag,2]]],4],m2++,
    For[m21=0,m21<=Min[basestates[[flag,3]],5],m21++,

For[n21=innerflag[basestates[[flag,1]],basestates[[flag,2]],
basestates[[flag,4]],basestates[[flag,5]],m1, m2,
m21,ps],n21<=
maxn21[basestates[[flag,1]],basestates[[flag,2]],basestates[[
flag,4]],basestates[[flag,5]],m1,m2,m21,ps],n21++,
  i=basestates[[flag,1]]+basestates[[flag,4]]-m1-
m21+n21;
  j=basestates[[flag,2]]+basestates[[flag,5]]-m2-
n21;
  k=basestates[[flag,3]]-m21+n21;
  v2=If[s1-basestates[[flag,1]]-
basestates[[flag,4]]<= 0,0,s1-basestates[[flag,1]]-
basestates[[flag,4]]];
  w2=If[s2-basestates[[flag,2]]-
basestates[[flag,5]]<= 0,0,s2-basestates[[flag,2]]-
basestates[[flag,5]]];
  v1=v2;
  w1=w2;
  target={
    {i, j, k, v1, w1}
  };
  If[Head[locator[i,j,k,v1,w1]]!=
Integer,Break[]];basestates=AppendTo[basestates,Part[target,
1]];

```



```

locator[i, j, k, v1, w1]=Length[basestates]
]
]
];Monitor[completion2=flag, completion2];
]];
Print["s1=", s1, " s2=", s2, " ps=", ps];
Print["Number of unique states = ", Length[basestates]];
numberofcells=Length[basestates];

(*Next we set up a sparse array for the transition
probability matrix and fill it with the appropriate values*)
Do[
Print["Doing P21= ", p21];
Print["Number of states ", Length[basestates]];

A=SparseArray[Table[0, {a, numberofcells}, {b, numberofcells}]];
For[row=1, row<=numberofcells, row++,

oldi=basestates[[row, 1]]; oldj=basestates[[row, 2]]; oldk=basestates[[row, 3]];
oldv1=basestates[[row, 4]]; oldw1=basestates[[row, 5]];
q1=n1-oldk+Min[0, oldi];
q2=n2+Min[0, oldj];
q21=oldk;
For[m1=0, m1<=Min[n1-oldk+Min[0, oldi], 4], m1++,
For[m2=0, m2<=Min[n2+Min[0, oldj], 4], m2++,
For[m21=0, m21<=Min[oldk, 5], m21++,
(*start=innerflag[oldi, oldj, oldv1, oldw1, m1, m2,
m21, ps]; *)
nX21=maxn21[oldi, oldj, oldv1, oldw1, m1, m2, m21, ps];
For[n21=innerflag[oldi, oldj, oldv1, oldw1, m1, m2,
m21, ps], n21<=nX21, n21++,
i=oldi+oldv1-m1-m21+n21;
j=oldj+oldw1-m2-n21;
k=oldk-m21+n21;
v2=If[s1-basestates[[row, 1]]-
basestates[[row, 4]]<= 0, 0, s1-basestates[[row, 1]]-
basestates[[row, 4]]];
w2=s2-basestates[[row, 2]]-basestates[[row, 5]];
v1=v2;
w1=w2;
If[Head[locator[i, j, k, v1, w1]]≠
Integer, Break[], column=locator[i, j, k, v1, w1]];

A[[row, column]]=A[[row, column]]+grandBinomial[m1, q1, m2, q2, m2
1, q21, n21, nX21, ps];

]
]
]

```

```

]Monitor[completion=row, completion];
];
(*Next we validate the matrix by testing that the rows
all sum to 1*)
Rowsum=Table[0, {numberofcells}];
numberofcells

$$\sum_{y=1} A[[x, y]]$$

Do[Rowsum[[x]]=
, {x, 1, numberofcells}];
Print["Check on row sums = ", (
numberofcells

$$\sum_{kount1=1} \text{Rowsum}[[kount1]]$$

)/numberofcells];
(*Next we manipulate the matrix to create the set of
simultaneous equations for determining the value of PI
variables*)
 $\psi$ =Table[ $\Pi$ [kount1], {kount1, 1, Length[Rowsum]}];
 $\phi$ =ReplacePart[ $\psi$ , numberofcells $\rightarrow$ 1];
unity=Table[1, {kount1, 1, numberofcells}];
AT=Transpose[A]; Clear[A];
CF=ReplacePart[AT, Part[numberofcells] $\rightarrow$ unity];
Clear[AT];
equations=Thread[CF. $\psi$ == $\phi$ ]; Clear[ $\phi$ , CF];
Prules=Solve[equations,  $\psi$ ]; Clear[ $\psi$ ];
PI=Table[ $\Pi$ [kount1] /.
Prules, {kount1, 1, Length[Rowsum]}];
I1:=0; I2:=0; IT=0;
For
[a=1, a<numberofcells, a++, I1=If[Part[basestates[[a]], 1]>0, I1+
Part[basestates[[a]], 1]*Part[PI[[a]], 1], I1]]; I1;
For
[a=1, a<numberofcells, a++, I2=If[Part[basestates[[a]], 2]>0, I2+
Part[basestates[[a]], 2]*Part[PI[[a]], 1], I2]]; I2;

V1:=0; V2:=0; W1:=0; W2:=0;
For
[a=1, a<numberofcells, a++, V1=V1+Part[basestates[[a]], 4]*Part[
PI[[a]], 1]];

For
[a=1, a<numberofcells, a++, W1=W1+Part[basestates[[a]], 5]*Part[
PI[[a]], 1]];

 $\Delta$ =0;
For
[a=1, a<numberofcells, a++,  $\Delta$ = $\Delta$ +Part[basestates[[a]], 3]*Part[P
I[[a]], 1]];  $\Delta$ ;
E1=0; For
[a=1, a<numberofcells, a++, E1=If[Part[basestates[[a]], 1]<0, E1+

```

```

Part[PI[[a]], 1], E1]; E1;
    E2=0; For
[a=1, a<=numberofcells, a++, E2=If[Part[basestates[[a]], 2]<0, E2+
Part[PI[[a]], 1], E2]; E2;
    E12=0; For
[a=1, a<=numberofcells, a++, E12=If[Part[basestates[[a]], 1]<0||P
art[basestates[[a]], 2]<0, E12+Part[PI[[a]], 1], E12]; E12;
    F1=1-E1; F2=1-E2; F12=1-E12;
    B1=0; For
[a=1, a<=numberofcells, a++, B1=If[Part[basestates[[a]], 1]<0, B1-
Part[basestates[[a]], 1]*Part[PI[[a]], 1], B1];
    B2=0; For
[a=1, a<=numberofcells, a++, B2=If[Part[basestates[[a]], 2]<0, B2-
Part[basestates[[a]], 2]*Part[PI[[a]], 1], B2];

AppendTo[results, {I1, I2, Δ, V1, W1, F1, F2, F12, B1, B2, s1, s2, ps, p21
}]; Export["excelresultswithorders106.xls", results];

',
{p21, {.05, .1}}]; Export["excelresultswithorders105.xls", resul
ts],
    {ps, 1, 1}}
',
    {s1, 4, 4}};,
    {s2, 2, 2}}];

```

```

excelresults=results;
results=Drop[results, 1];
resultslength=Length[results]

```

```

Export["substitutionoutputwithorders105.XLS", excelresults, "X
LS"]

```

substitutionoutputwithorders105.XLS

Next we prepare the plotting data

```

numberofcells
    ∑ Part[PI[[e]], 1]
check=

```

```

1.
stoptime=DateString[];

```

```

processtime=AbsoluteTime[stoptime]-AbsoluteTime[starttime];
Print["Total Evaluation Time = ", processtime, " seconds"]
"Total Evaluation Time = " 7372 " seconds"

```

APPENDIX B
MATHEMATICA CODE FOR THE GENERAL
SUBSTITUTION MODEL

```
ClearAll["Global`*"]
```

First we develop the base matrix of available states. In order to facilitate later manipulation and optimization we will program the possible cells as functions of the form state[i, j, k]

```
datatable={};
datatable=AppendTo[datatable,Part[({
  {"sa", "sb", "sc", "m", "n", "λA", "λB", "μA", "μB", "μC",
  "PA", "PB", "fillrateT", "inventoryA", "inventoryB", "inventoryC",
  "backorderA", "backorderB", "fillrateA", "fillrateB"}
}),1]];

Do[Do[Do[Do[Do[
  m= 30;n= 30;PB=PA;count=0;counter=0;
  λB=. 6;μA=. 1;μB=. 1;λA=0. 3;
  elements =(sa+m+1) (sb+n+1) (sc+1);

  Print["elements=",elements];

  unknowns={};
  sumtol=" ";

  For [i=-m,i≤sa,i++,
    For [j=-n,j≤sb,j++,
      For [k=0,k≤sc,k++,

        If[i==sa&&j==sb&&k==sc, sumtol=sumtol<>ToString[P[i, j, k]]<>"=1",
          sumtol=sumtol<>ToString[P[i, j, k]]<>"+"];
        ];
      ];
    ];
  sumtol=ToExpression[sumtol];
  For[i=-m-1,i≤sa+1,i++,
    For[j=-n-1,j≤sb+1,j++,
      For[k=-1,k≤sc+1,k++,
        pUP[i, j, k]=0;
        pDOWN[i, j, k]=0;
        pIN[i, j, k]=0;
        pOUT[i, j, k]=0;
        pRIGHT[i, j, k]=0;
        pLEFT[i, j, k]=0;
        ];
      ];
    ];
  For [i=-m,i≤sa,i++,
    For [j=-n,j≤sb,j++,
      For [k=0,k≤sc,k++,
        pa[i, j, k]=Which[k==0, 0, i>0, 0, True, PA];
        pb[i, j, k]=Which[k==0, 0, j>0, 0, True, PB];
        pUP[i, j, k]=(sa-i)*μA;
        pLEFT[i, j, k]=(sb-j)*μB;
        pOUT[i, j, k]=(sc-k)*μC;
        pDOWN[i, j, k]=Which[i==m, 0, i>0, λA, k==0, λA, True, λA (1-
pa[i, j, k])];
        pRIGHT[i, j, k]=Which[j==n, 0, j>0, λB, k==0, λB, True, λB (1-
pb[i, j, k])];
        pIN[i, j, k]=Which[i≤0 && j>0 &&k>0, λA*pa[i, j, k], i>0 && j ≤
0 && k>0, λB*pb[i, j, k], i≤0 && j≤0 &&
```

```

k>0,λA*pa[i,j,k]+λB*pb[i,j,k],True,0];

      count=count+1;
      (*Print[count, {"i,",",j,",",k,"}
", "pUP=",pUP[i,j,k], " pDOWN=",pDOWN[i,j,k], " pIN=",pIN[i,j,k], "
pOUT=",pOUT[i,j,k], " pRIGHT=",pRIGHT[i,j,k], "
pLEFT=",pLEFT[i,j,k], " pa=",pa[i,j,k], " pb=",pb[i,j,k]]*)

    ];
  ];
];
(*Now we have set all the possible states and the movement
rules for leaving each state.
We are going to enforce the condition that, at steady state,
the rate of movement into a state is exactly equal to the rate of
movement out of that state. This will give us a number of equations
equal to the number of unknowns where the unknowns are the set of
probabilities for being in each state. By combining these rules
with the fact that there is exactly a probability of 1 that we will
be in some state we can enforce a single unique solution set for the
probabilities.
We are going to cycle through P[i, j, k] and output a
equation for each unique combination of i, j, k in the domain of all
sets. We will create these equations as text and then after all
done convert the text to an expression so that we may solve the
family of equations.*)
count=0;
For [i=-m,i≤sa,i++,
  For [j=-n,j≤sb,j++,
    For [k=0,k≤sc,k++,
      count=count+1;
      AppendTo[unknowns,P[i,j,k]];

eqn[count]=P[i,j,k]*(pUP[i,j,k]+pDOWN[i,j,k]+pIN[i,j,k]+pOUT[i,j,k]+
pRIGHT[i,j,k]+pLEFT[i,j,k])= P[i-1,j,k]*pUP[i-
1,j,k]+P[i+1,j,k]*pDOWN[i+1,j,k]+P[i,j,k+1]*pIN[i,j,k+1]+P[i,j,k-
1]*pOUT[i,j,k-1]+P[i,j+1,k]*pRIGHT[i,j+1,k]+P[i,j-1,k]*pLEFT[i,j-
1,k];

    ];
  ];
];
equations=Table[eqn[kount],{kount,1,elements}];
Drop[equations,1];
AppendTo[equations,sumt01];
MatrixForm[%];
unknowns;
probabilities=Solve[equations,unknowns];
SteadyState=Table[unknowns[counter] /.
probabilities,{counter,1}];
MatrixForm[%];
inventoryA=0;
inventoryB=0;
inventoryC=0;
backorderA=0;
backorderB=0;
fillrateA=0;
fillrateB=0;
fillrateT=0;

```

```

For [i=-m, i≤sa, i++,
  For [j=-n, j≤sb, j++,
    For [k=0, k≤sc, k++,
      inventoryA=inventoryA+If[i>0, i*P[i, j, k] /.
probabilities, 0];
      inventoryB=inventoryB+If[j>0, j*P[i, j, k] /.
probabilities, 0];
      inventoryC=inventoryC+If[k>0, k*P[i, j, k] /.
probabilities, 0];
      backorderA=backorderA+If[i≤0, -i*P[i, j, k] /.
probabilities, 0];
      backorderB=backorderB+If[j≤0, -j*P[i, j, k] /.
probabilities, 0];
      fillrateA=fillrateA+If[i≥0, P[i, j, k] /. probabilities, 0];
      fillrateB=fillrateB+If[j≥0, P[i, j, k] /. probabilities, 0];
      fillrateT=fillrateT+If[j≥0 && i≥0, P[i, j, k] /.
probabilities, 0];

    ];
  ];
];
inventoryC=If[sc==0 || PA==0, {0}, inventoryC];
outputvector=Part[({
  {sa, sb, sc, m, n, λA, λB, μA, μB, μC, PA, PB,
Part[fillrateT, 1], Part[inventoryA, 1], Part[inventoryB, 1],
Part[inventoryC, 1], Part[backorderA, 1], Part[backorderB, 1],
Part[fillrateA, 1], Part[fillrateB, 1]}
}), 1];

datatable=AppendTo[datatable, outputvector]; Print["sb=", sb];, {sb, 3, 8}
]; Export["generaldatasc5part2.xls", datatable]; Print["sa=", sa];, {sa, 3
, 8}]; Print["sc=", sc];, {sc, 1, 3}]; Print["PA=", PA];, {PA, .5, 1, .5}]; Print
["μC=", μC];, {μC, {.2}}];

Export["generaldatapart2copy.xls", datatable2];

```